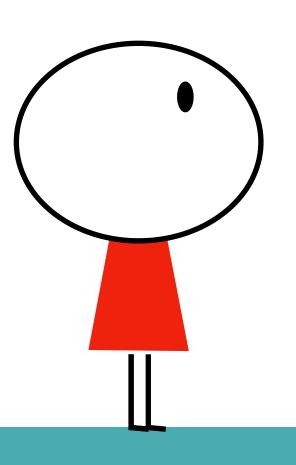
# Lattice-based crypto

# Overview of attack techniques and countermeasures

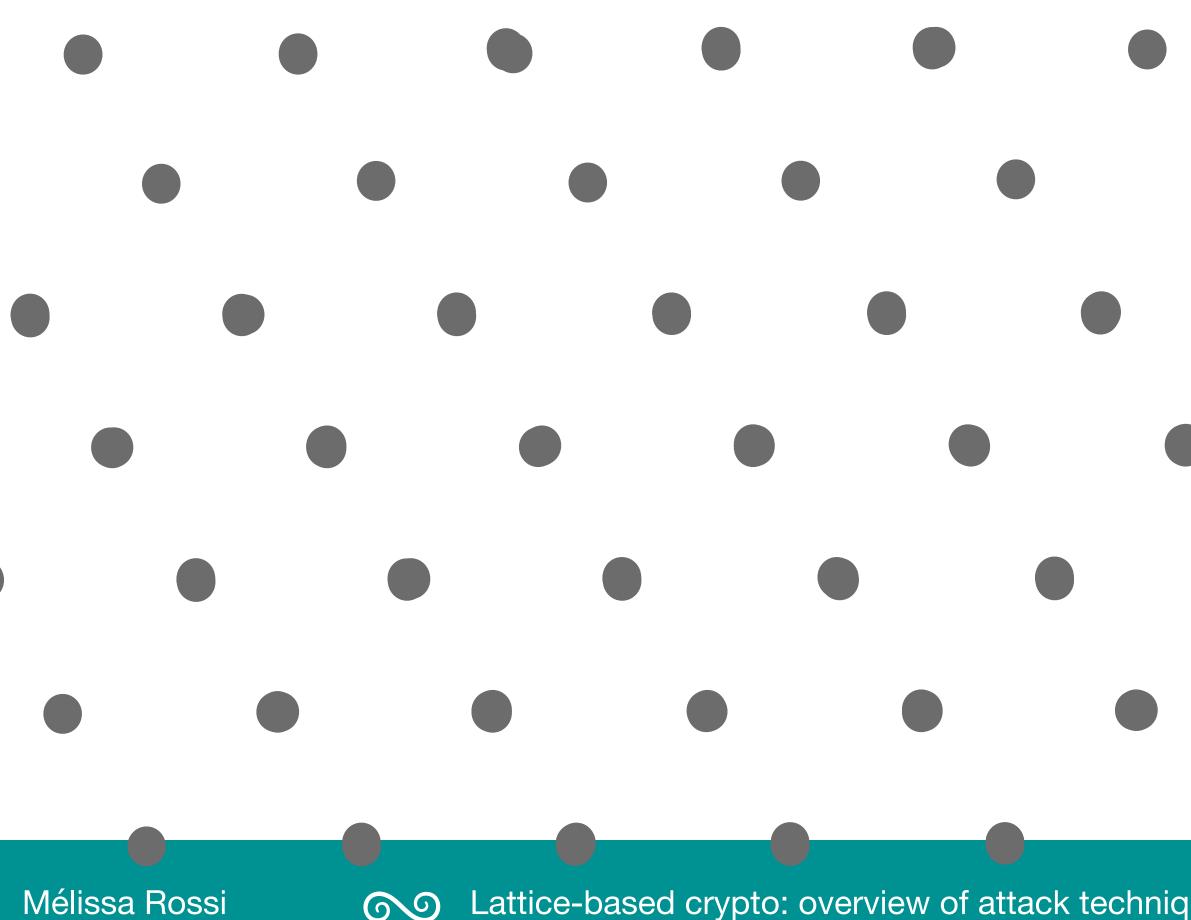






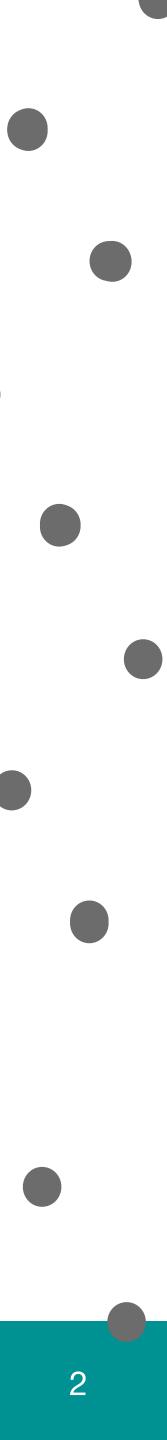


A lattice  $\Lambda$  is an additive subgroup generated by nlinearly independent vectors of  $\mathbb{R}^n$ .

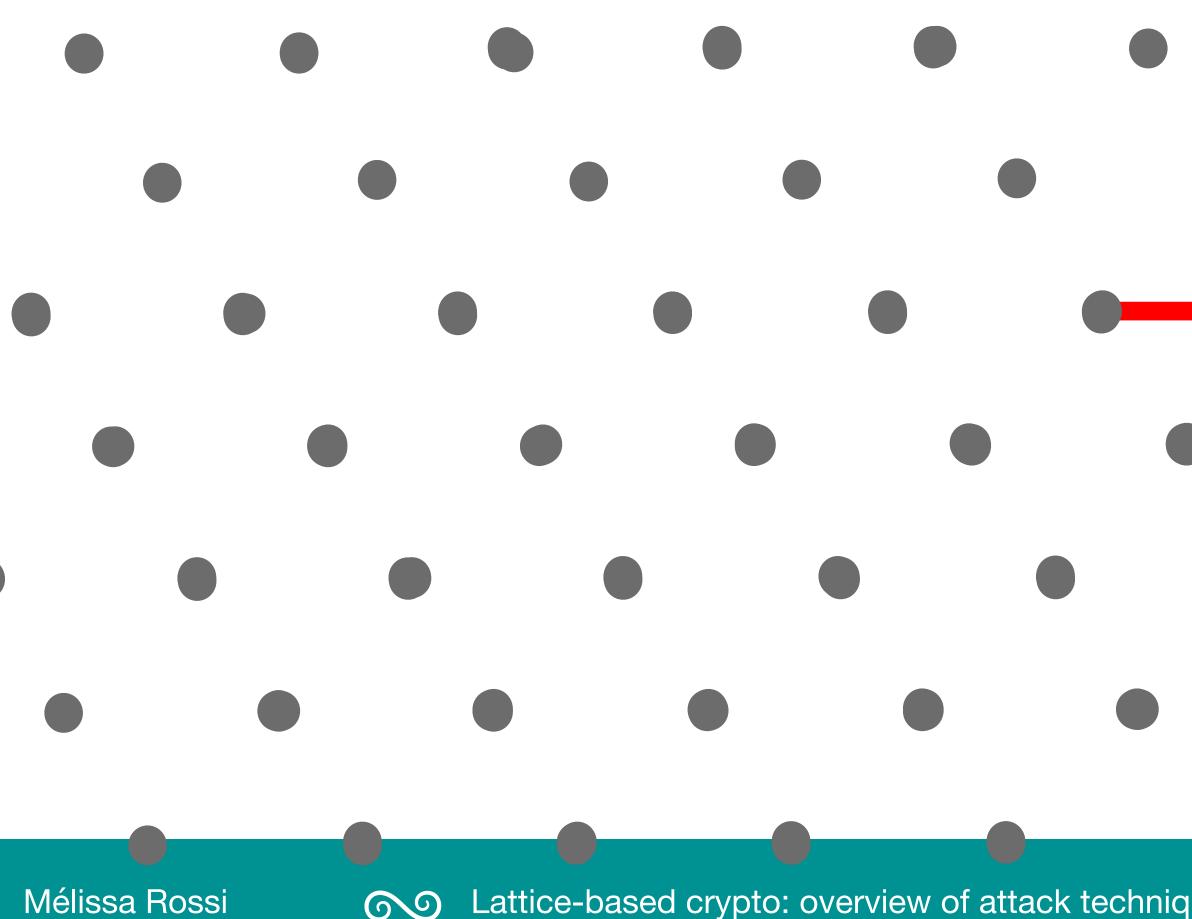


# Lattices and hard problems

So Lattice-based crypto: overview of attack techniques and countermeasures So

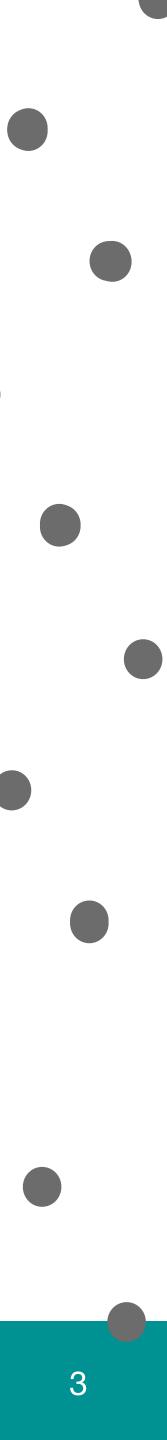


A lattice  $\Lambda$  is an additive subgroup generated by nlinearly independent vectors of  $\mathbb{R}^n$ .



# Lattices and hard problems

So Lattice-based crypto: overview of attack techniques and countermeasures So



# Lattices and hard problems

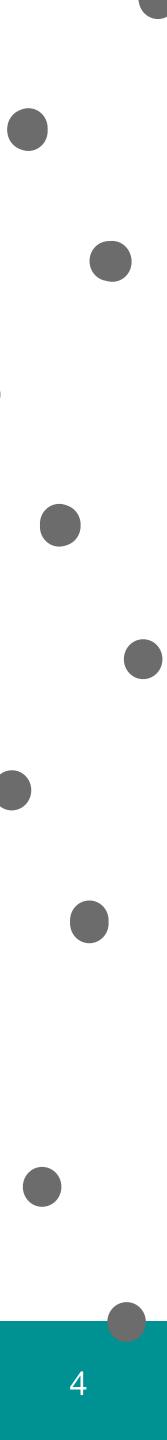
Given a lattice  $\Lambda$ 

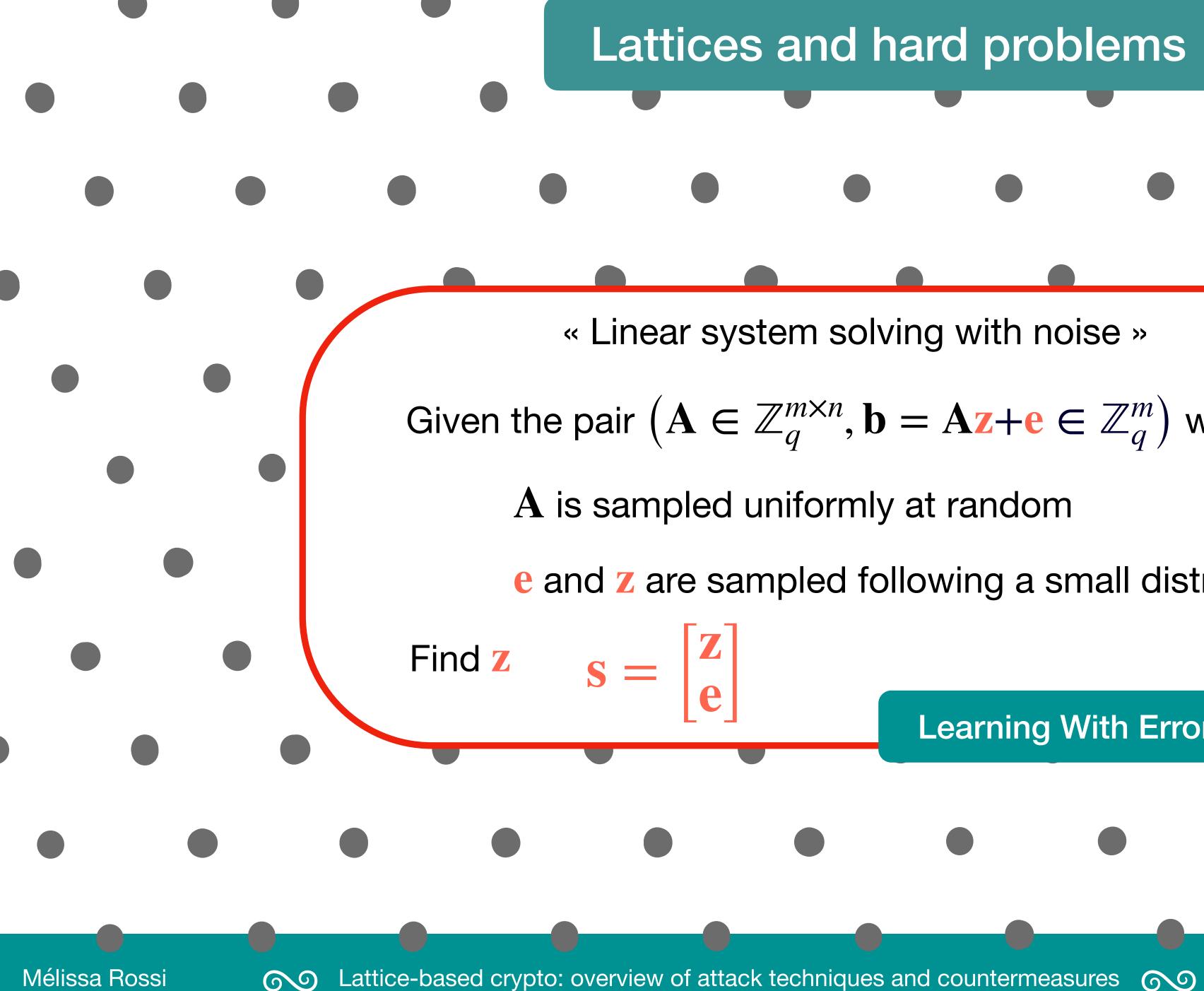
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Find the vector **v** that has the smallest nonzero norm

Short Vector Problem (SVP)





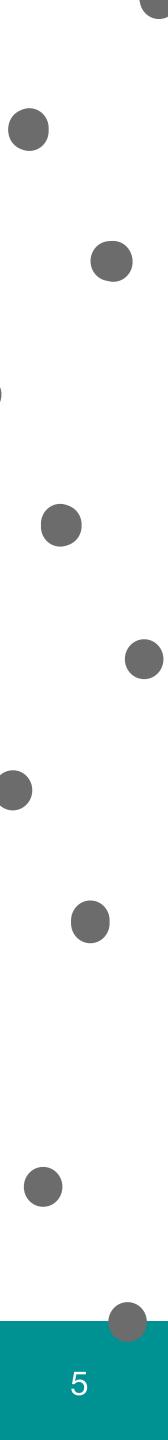
# Lattices and hard problems

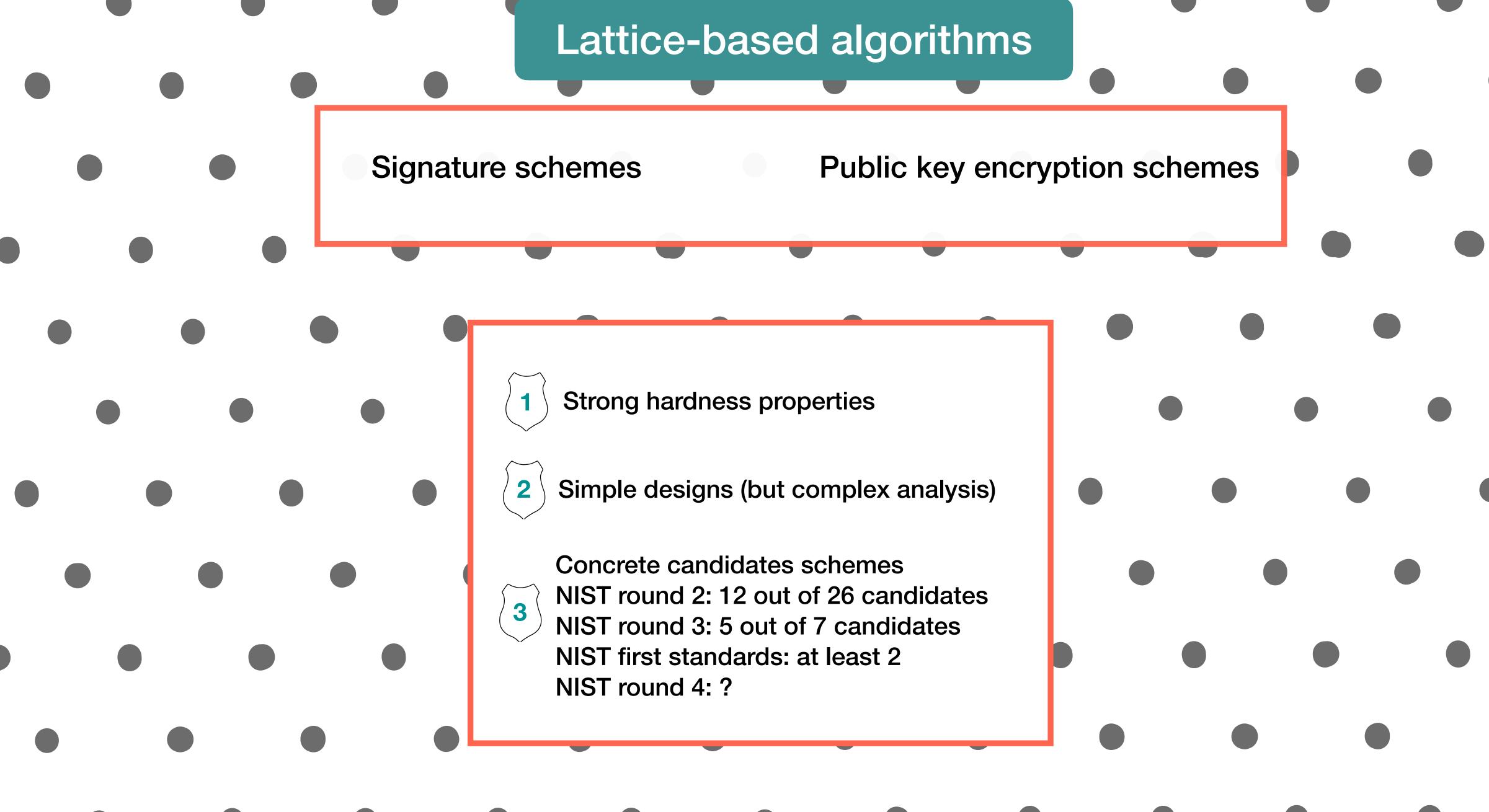
« Linear system solving with noise »

, 
$$\mathbf{b} = \mathbf{A}\mathbf{z} + \mathbf{e} \in \mathbb{Z}_q^m$$
 where

e and z are sampled following a small distribution  $\chi$ 

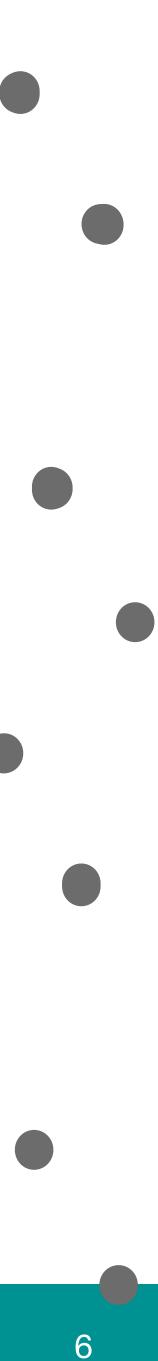
Learning With Errors (LWE)





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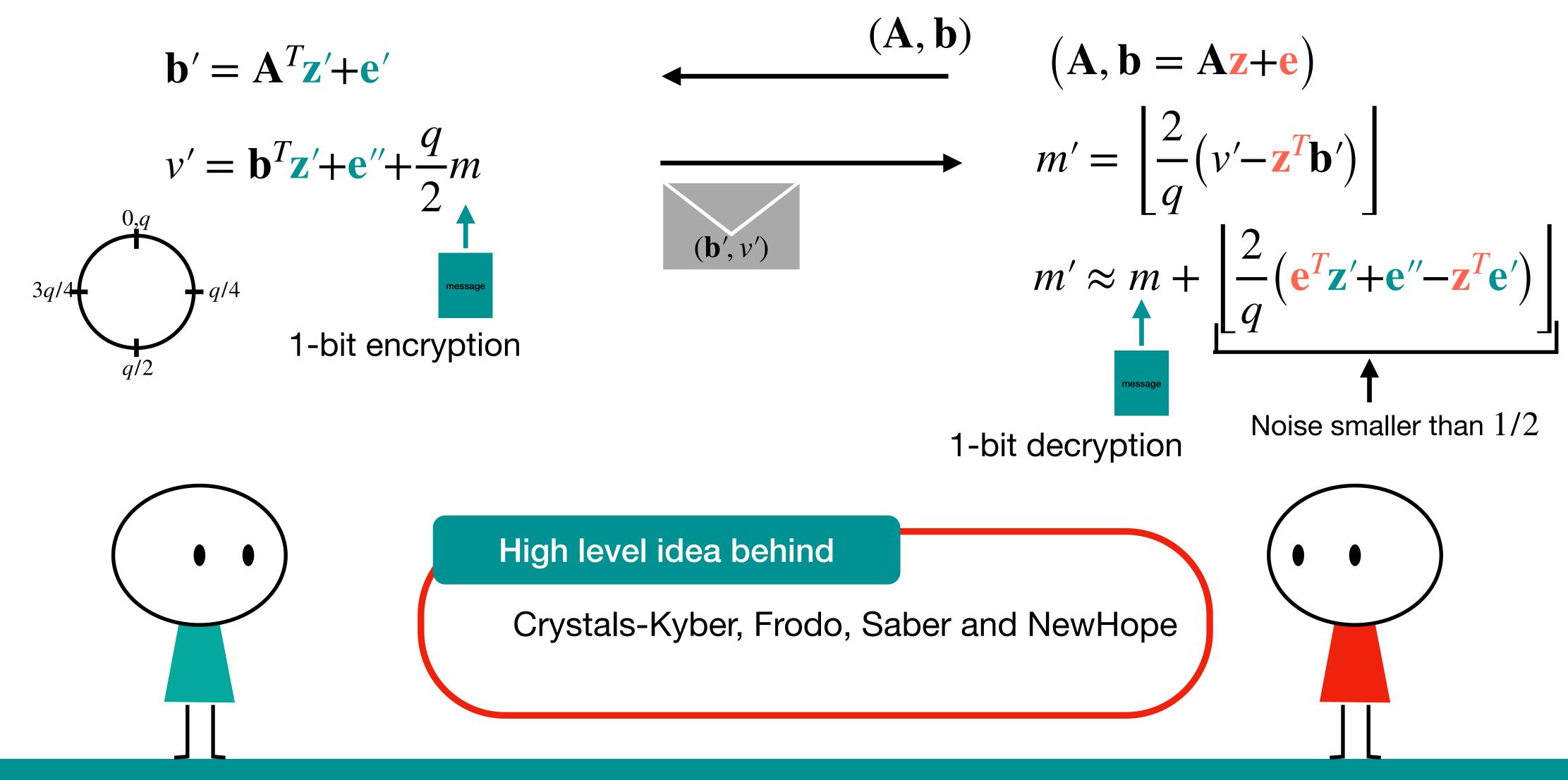
So Lattice-based crypto: overview of attack techniques and countermeasures So



J. Ding, X. Xie and X. Lin EUROCRYPT'14

### C. Peikert PQCRYPTO'14

- J. W. Bos, C. Costello, M. Naehrig and D. Stebila S&P'15
- E. Alkim, L. Ducas, T. Pöppelmann and P. Schwabe USENIX'16



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So Lattice-based crypto: overview of attack techniques and countermeasures So

# LWE-based public key encryption in a nutshell

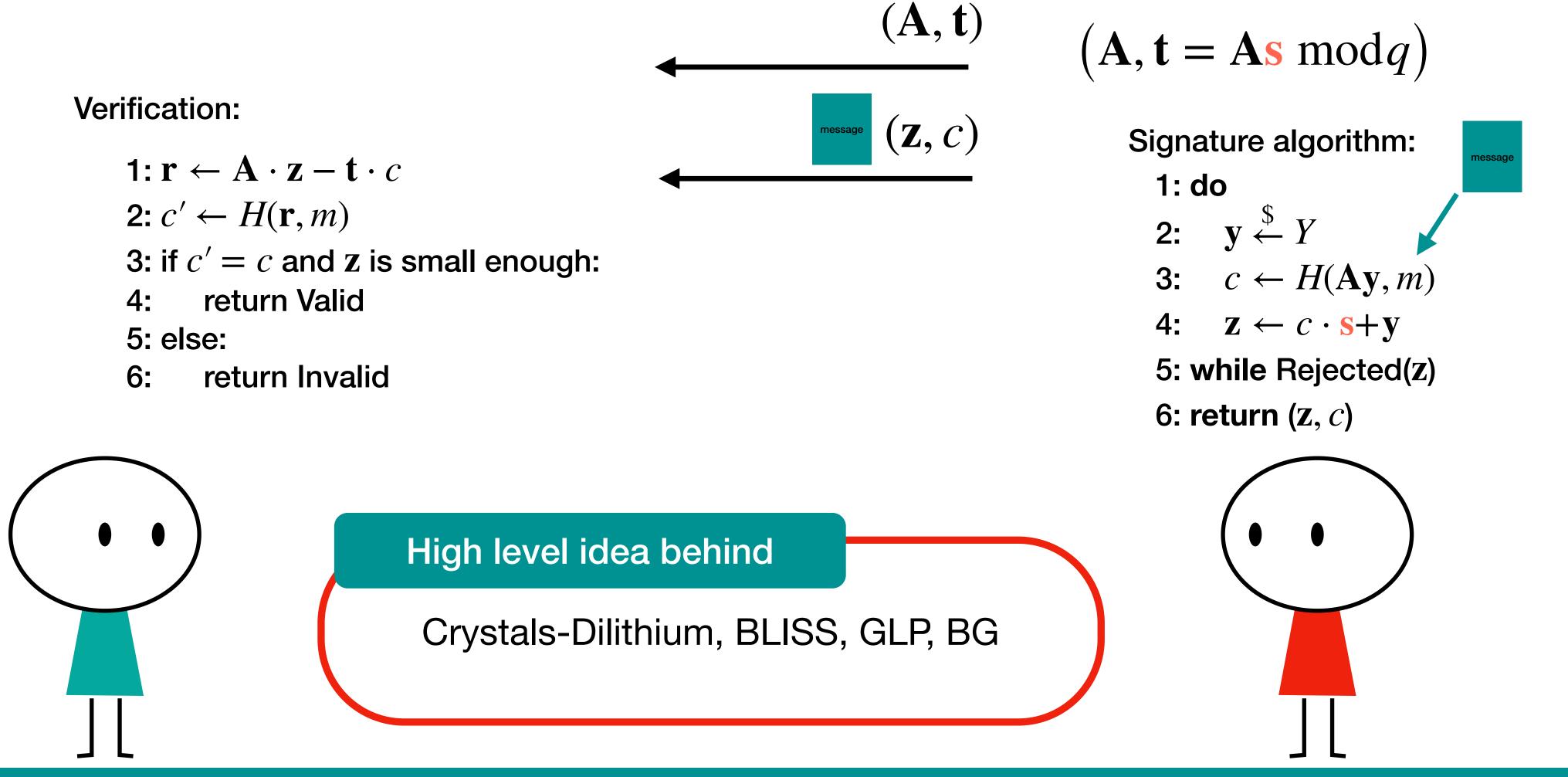




# A Fiat-Shamir with aborts signature in a nutshell

- V. Lyubashevsky EUROCRYPT'12
- L. Ducas, A. Durmus, T. Lepoint and V. Lyubashevsky CRYPTO'13
- S. Bai and D. Galbraith CT-RSA'14

**2**:  $c' \leftarrow H(\mathbf{r}, m)$ 3: if c' = c and z is small enough: return Valid 4: 5: else:



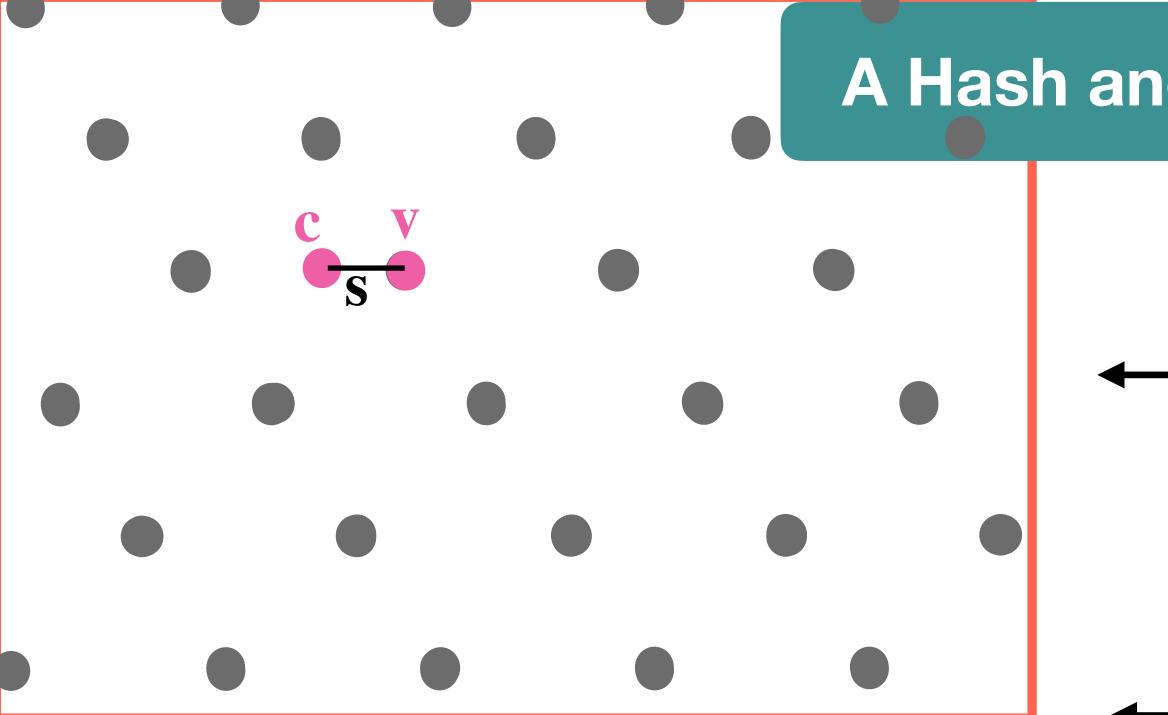
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Short Integer Solution (SIS)

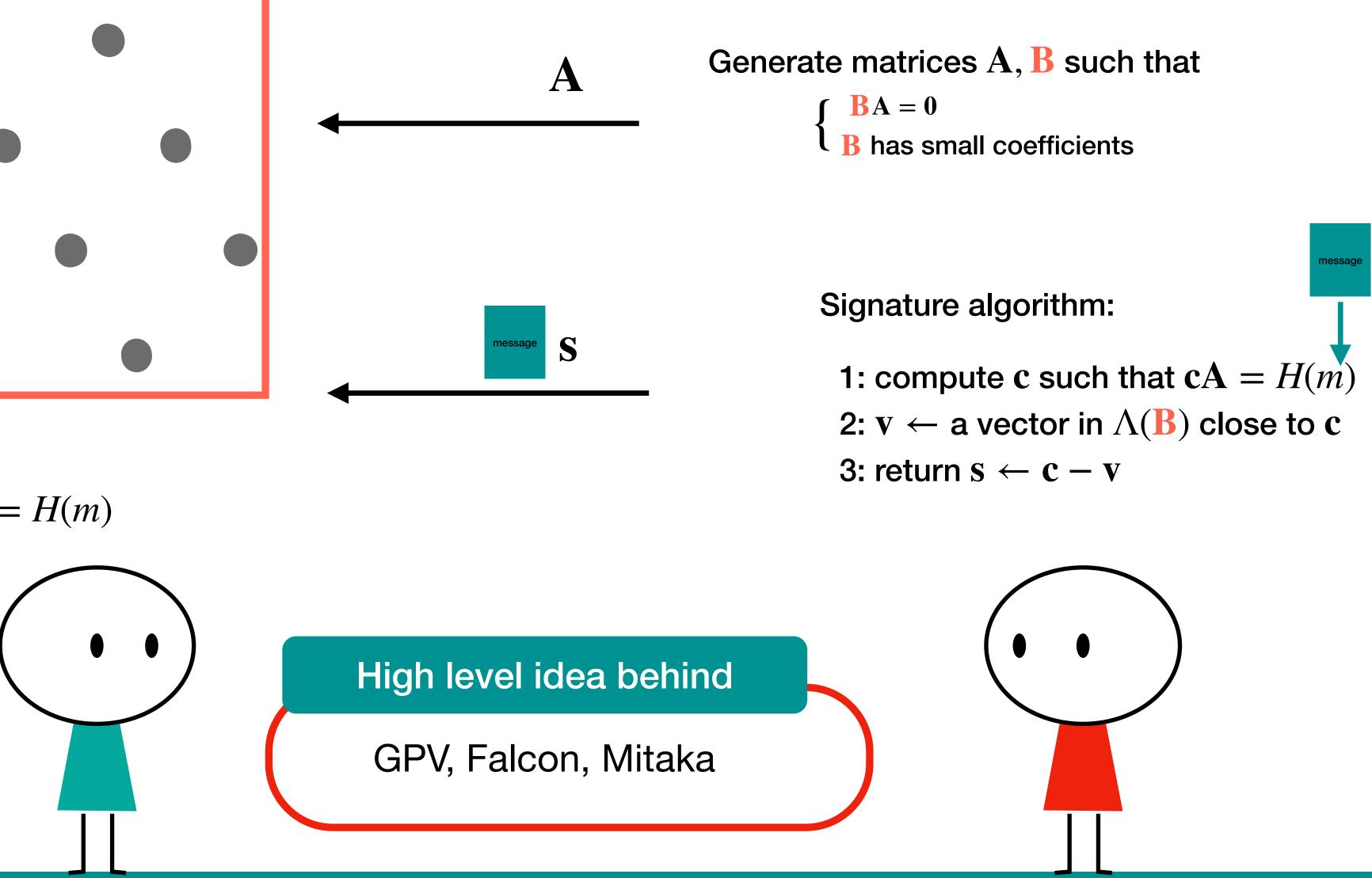






Verification:

- 1: if s is short and sA = H(m)
- return Valid 2:
- 3: else:
- return Invalid 4:



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# A Hash and sign in a nutshell

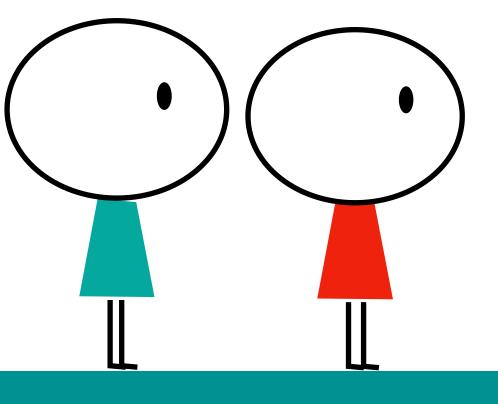


9



## Signature schemes

## Public key encryption schemes



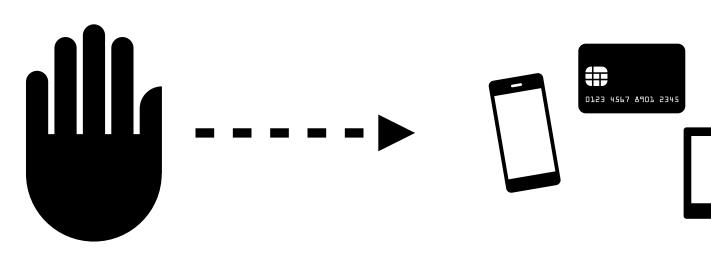
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# Lattice-based algorithms

Are you secure for real-world development?

Are you timing resistant? Are you secure against physical attacks? Are you misuse resistant? Are you decryption-failure resistant? ... by how much ?







Due to time limit, we will not go into much details during this overview. But, the purpose of C2 days is exchanging, so feel free to come and ask for details.

Countermeasure

3

techniques

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attack

ules and tools

 $\bigcirc \bigcirc$ 

Là

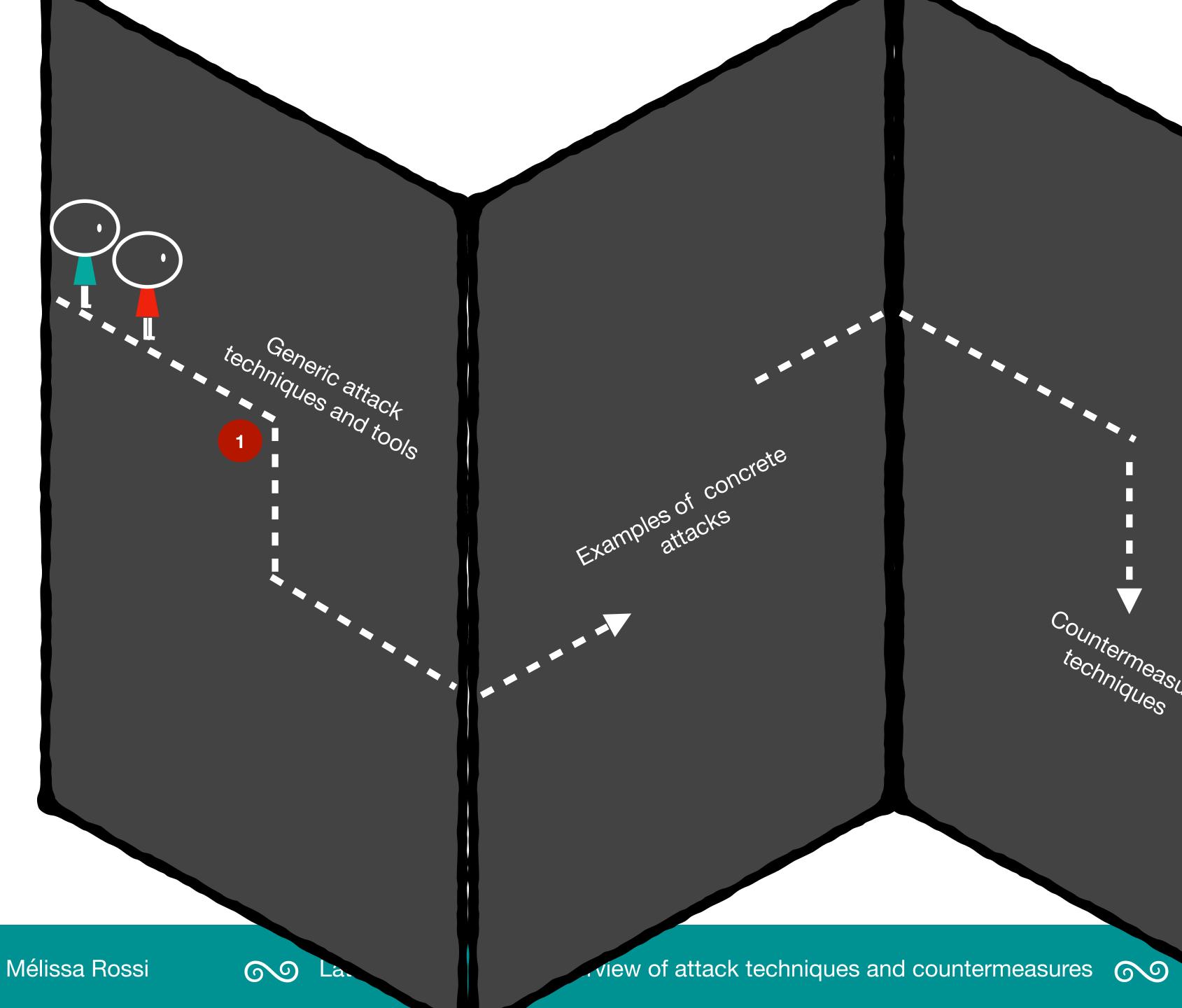
view of attack techniques and countermeasures 60

Examples of concrete attacks

## Disclaimer





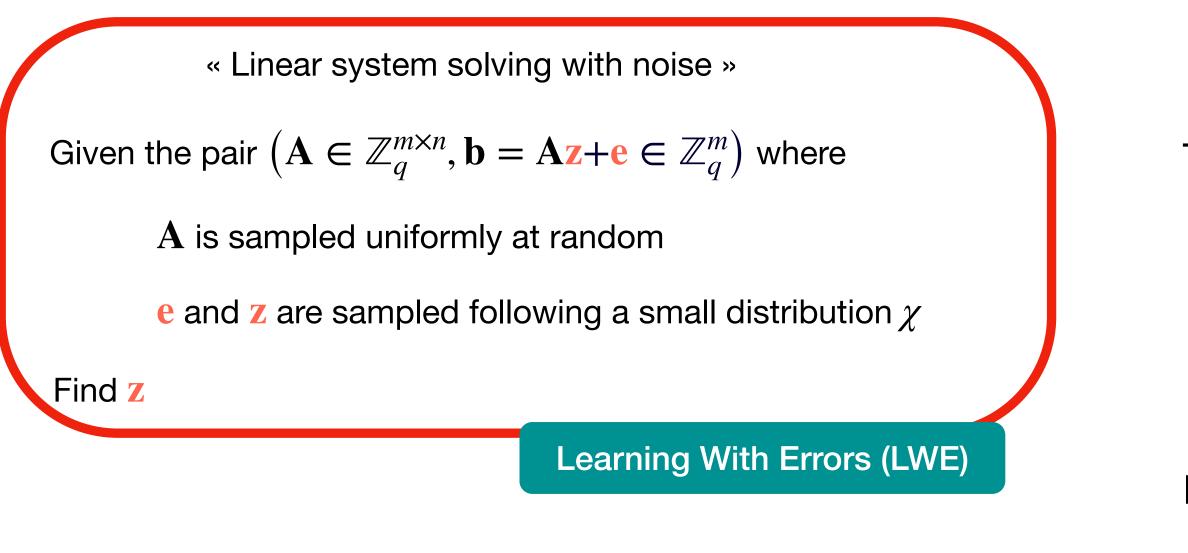


Countermeasure techniques





# Primal attack to assess the mathematical security





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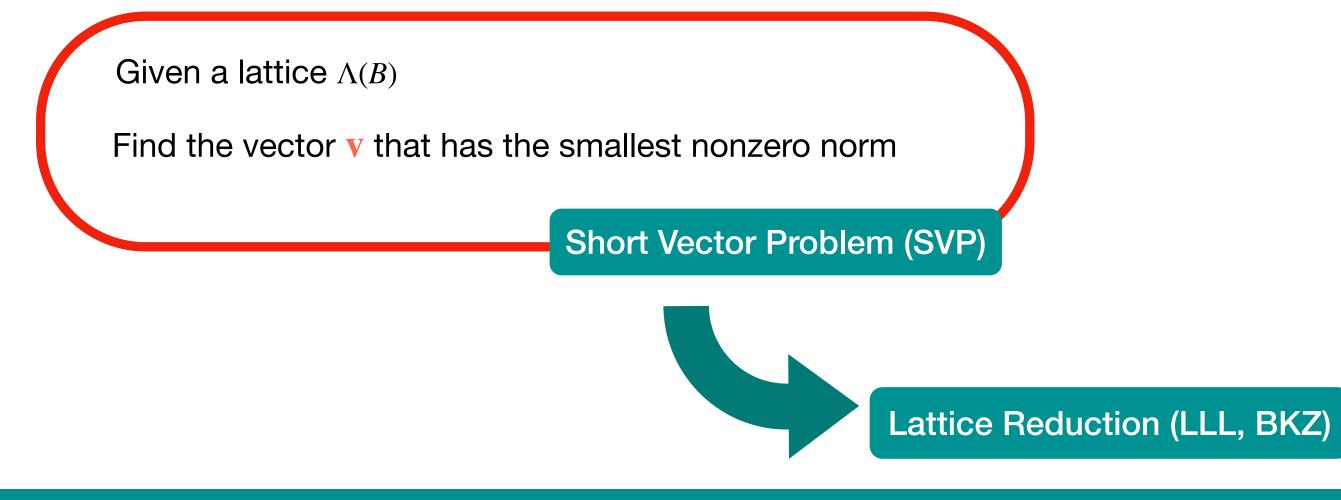
 $LWE \mapsto SVP \mapsto Lattice reduction$ 

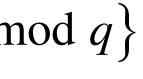
Kannan's embedding:

The vector  $[\mathbf{e}^T, \mathbf{z}^T, 1]$  is a short vector of the lattice  $\Lambda(\mathbf{B})$  where

$$\mathbf{B} = \begin{bmatrix} q\mathbf{I_m} & 0 & 0 \\ -\mathbf{A} & -\mathbf{I_n} & 0 \\ \mathbf{b} & 0 & 1 \end{bmatrix}$$

More precisely,  $\Lambda(\mathbf{B}) = \{(\mathbf{x}^T, \mathbf{y}^T, w) \text{ such that } \mathbf{x} + \mathbf{A}\mathbf{y} - w\mathbf{b} = \mathbf{0} \mod q \}$ 









# Tools to assess the mathematical security (primal attack)

Quantifying security: Cost of the best known attack against the underlying lattice hard problem for specific parameters e.g.  $2^{128}$ 

# $LWE \mapsto SVP \mapsto Lattice reduction$ block-size of BKZ $\mapsto$ cost models $\mapsto$ bit security

- E. Alkim, L. Ducas, T. Pöppelmann, and P. Schwabe. USENIX'2016
- M. R Albrecht, F. Göpfert, F. Virdia, and T. Wunderer. ASIACRYPT'2017
- M. R. Albrecht, B. R. Curtis, A. Deo, A. Davidson, R. Player, E. W. Postlethwaite, F. Virdia, and T. Wunderer. SCN'2018

## • LWE estimator: Tool to compute the bit security of any LWE-NTRU-based scheme.

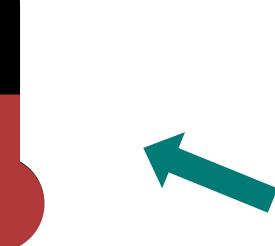
## • Leaky LWE estimator: Tool to include partial side information in the scale.

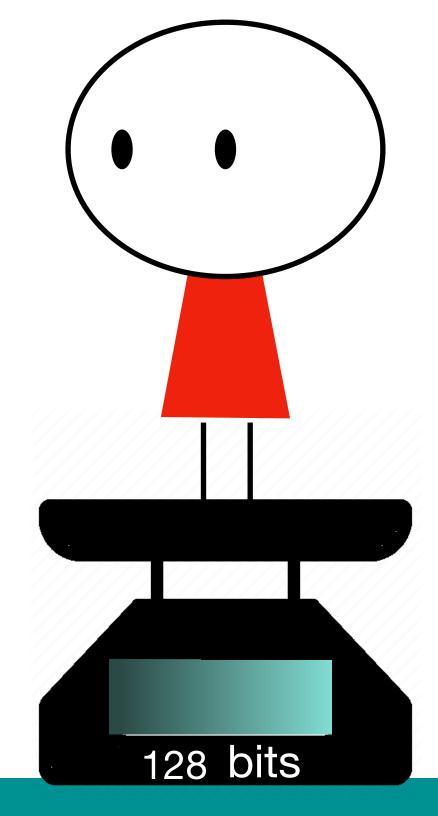
D. Dachman-Soled, L. Ducas, H. Gong and M. Rossi. CRYPTO'2020.

Side information Partial information on the secret (side-channel, timing) attacks, constraints on the design...)

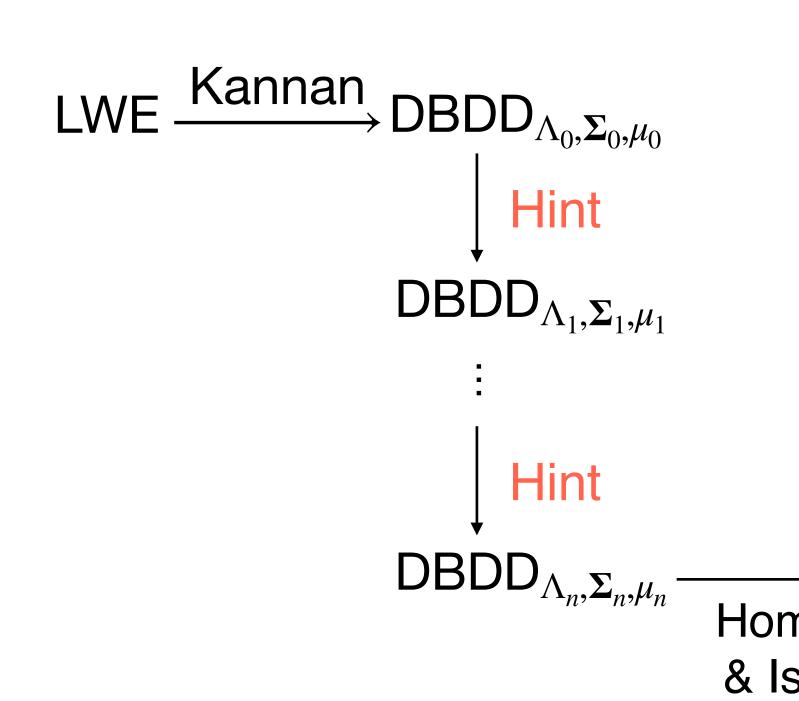
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**Perfect hints** 

$$\mathbf{v}, l \in \mathbb{Z}$$
 such that  $\langle \mathbf{s}, \mathbf{v} \rangle = l$ 

**Modular hints** 

 $\mathbf{v}, l, k \in \mathbb{Z}$  such that  $\langle \mathbf{s}, \mathbf{v} \rangle = l \mod k$ 

**Approximate hints** 

 $\mathbf{v}, \mathbf{l} \in \mathbb{Z}$  such that  $\langle \mathbf{s}, \mathbf{v} \rangle \approx \mathbf{l}$ 

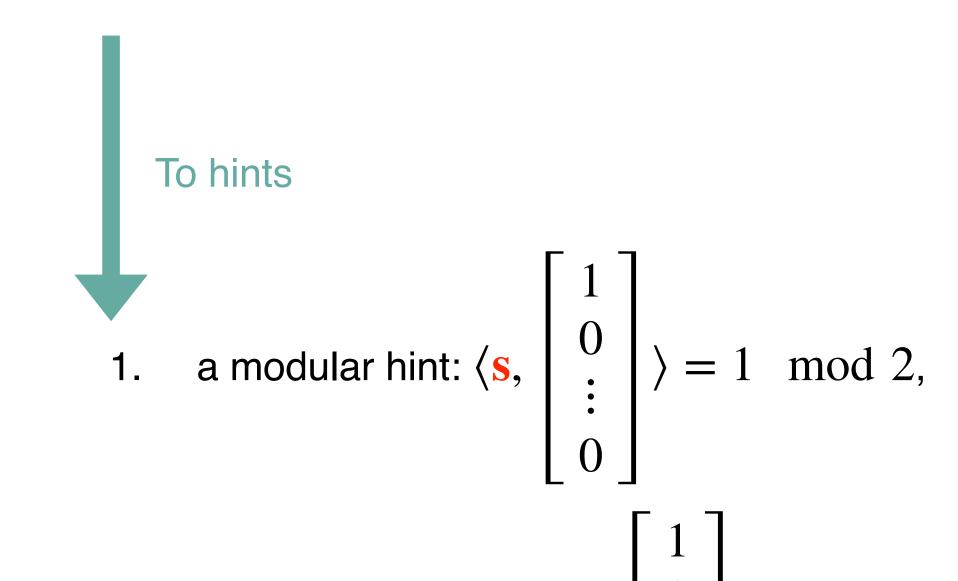
 $\rightarrow$  SVP  $\longrightarrow$  Lattice Reduction Homogenize & Isotropize





# Side-channel applications basic example

After a power analysis, attacker learns the hamming weight of  $s_0$ , say  $HW(s_0) = 2 \longrightarrow s_0 \in \{3, 5\}.$ 



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Secret coefficient  $s_i \in \{-5, ..., 5\}$  (represented by a signed 16-bits integer)

2. an approximate hint:  $\langle \mathbf{S}, \begin{bmatrix} 1\\0\\ \vdots\\0 \end{bmatrix} \rangle \approx 4$ , with variance 1.  $\mathbf{0}$ 



**Tool Demonstration** 

# Another attack: the hidden parallelepiped attack

## Generate matrices A, B such that

 $\mathbf{B}\mathbf{A} = \mathbf{0}$ 

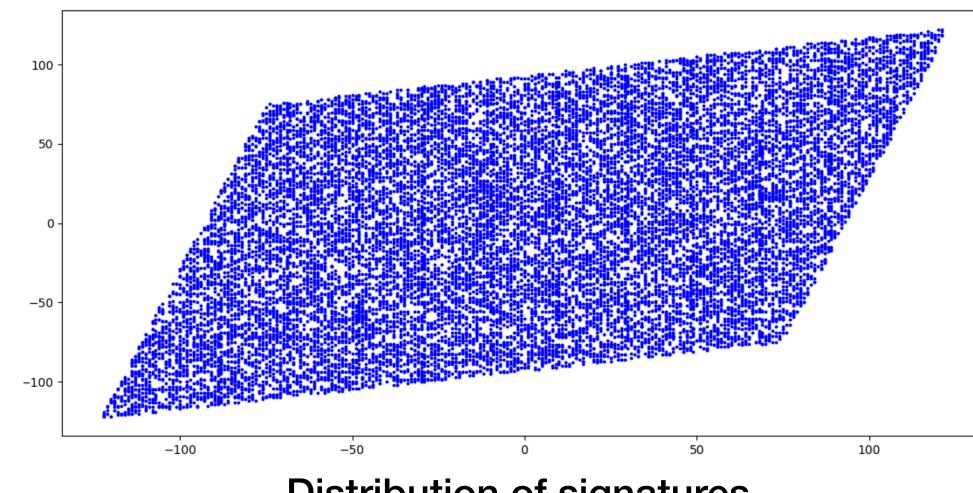
Signature algorithm:



- 2:  $\mathbf{v} \leftarrow \mathbf{a}$  vector in  $\Lambda(\mathbf{B})$  close to  $\mathbf{c}$
- 3: return  $s \leftarrow c v$



HPP attack: with enough signatures, we can « see » the private basis



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- ▶ P. Nguyen, O. Regev Eurocrypt'2006
- L. Ducas, P. Nguyen Asiacrypt'2012

has small coefficients

Babai's reduction: take the closest vector.



**Distribution of signatures** 

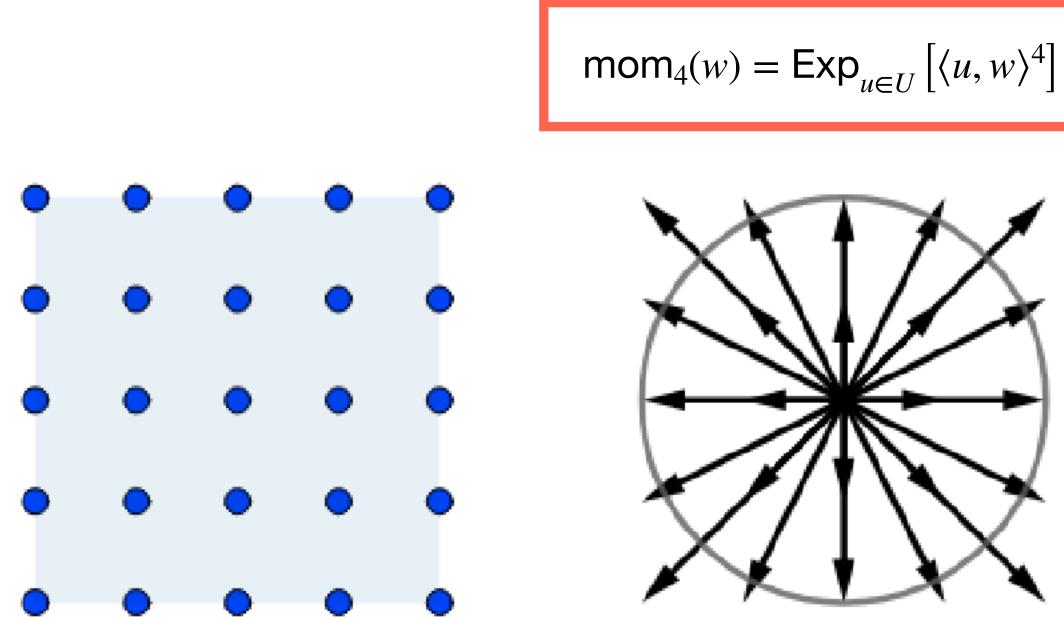




# Hidden parallelepiped attack: how to recover the basis?

Even if we see the basis in 2D, recovering the basis is tricky.

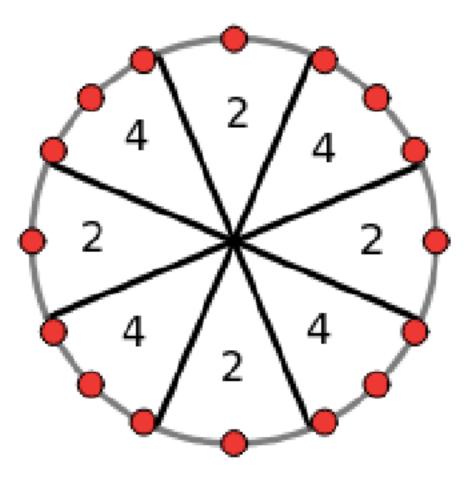
• First, we morph the problem into a hidden hypercube problem (Cholesky decomposition of the covariance matrix) Second, we note that the fourth moment over the unit sphere is minimized in the corners.



Thus, we can perform a gradient descent to recover the corners and then the basis.

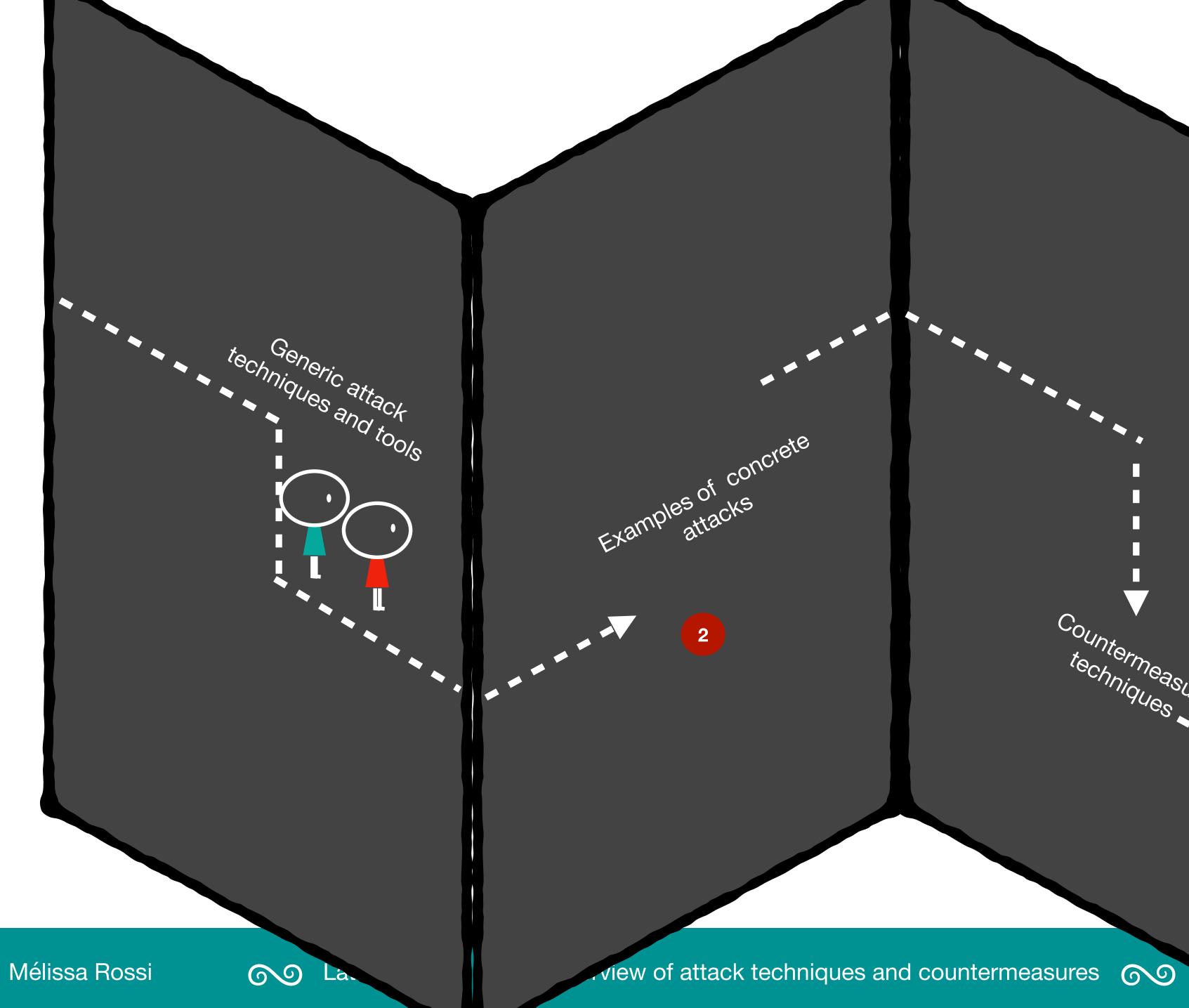
60 Lattice-based crypto: overview of attack techniques and countermeasures 60

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Countermeasure techniques







<u>Timing attack</u>: the attacker knows the time that the algorithm takes e.g. the number of iterations.

In lattice-based schemes, we always to sample small coefficients.



It implies computing transcendental functions exp(.) and cosh(.)

Many timing attacks targeting Gaussian distributions in lattice-based signature schemes

- L. Groot Bruinderink, A. Hülsing, T. Lange, and Y. Yarom. CHES'2016
- T. Espitau, P.-A. Fouque, B. Gérard, M. Tibouchi. SAC'2016
- P. Pessl, L. Groot Bruinderink, and Y. Yarom. ACM-CCS'2017

An example presented in the next slide

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# Timing attacks

Gaussians are often used for two reasons: Security reductions Performance

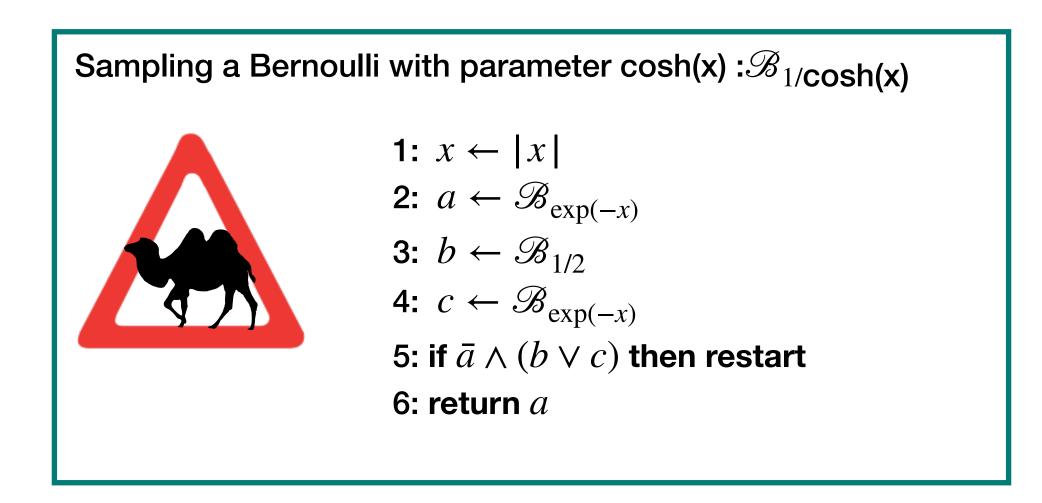
Hard to compute efficiently in constant time!

T. Espitau, P.-A. Fouque, B. Gérard and M. Tibouchi. ACM-CCS'2017 J. Bootle, C. Delaplace, T. Espitau, P.-A. Fouque and M. Tibouchi. ASIACRYPT'2018 G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi. ACM-CCS'2019 P.-A. Fouque, P. Kirchner, M. Tibouchi, A. Wallet, and Y. Yu. EUROCRYPT'2020





# An example of timing attack on BLISS signature scheme



Even if every Bernoulli sampling is constant time, there is still timing attack!

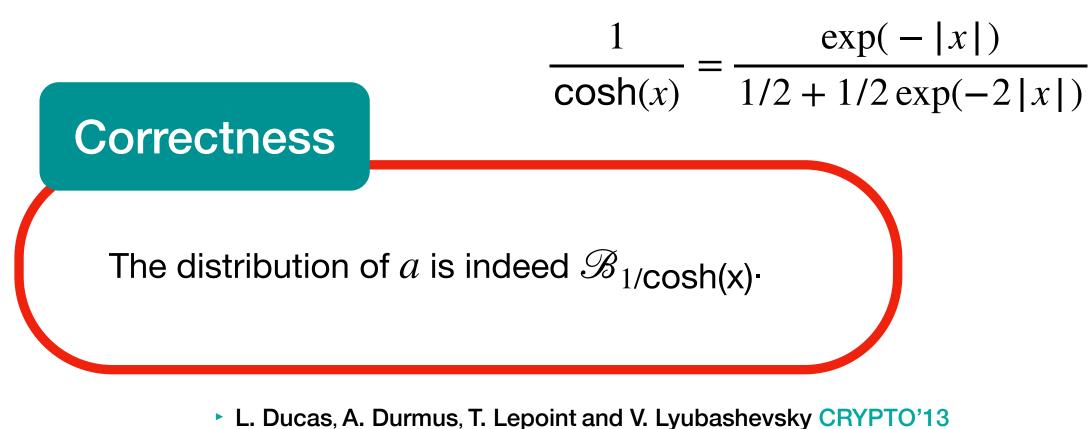
Probability of going from step 5 to step 6:

$$\mathbb{P}(\overline{a} \land (b \lor c)) = 1 - \mathbb{P}(\overline{a}) \cdot \mathbb{P}(b \lor c)$$

$$= 1 - (1 - \mathbb{P}(a)) \cdot (1 - \mathbb{P}(\overline{b} \land \overline{c}))$$

$$= 1 - (1 - \exp(-x)) \left(1 - \frac{1 - \exp(-x)}{2}\right)$$

$$= \frac{1 + \exp(-2x)}{2}$$
Depends on the input



## Idea of the attack

Actually 
$$x = - |\langle z, \mathbf{S}c \rangle|$$

We select the signatures (z, c) that end up in **one iteration**.

It means that 
$$\frac{1 + \exp(-2|\langle z, \mathbf{S}c \rangle|)}{2}$$
 is large.

Then,  $|\langle z, \mathbf{S}c \rangle|$  is close to 0.

Can be solved with a phase retrieval algorithm (machine learning).







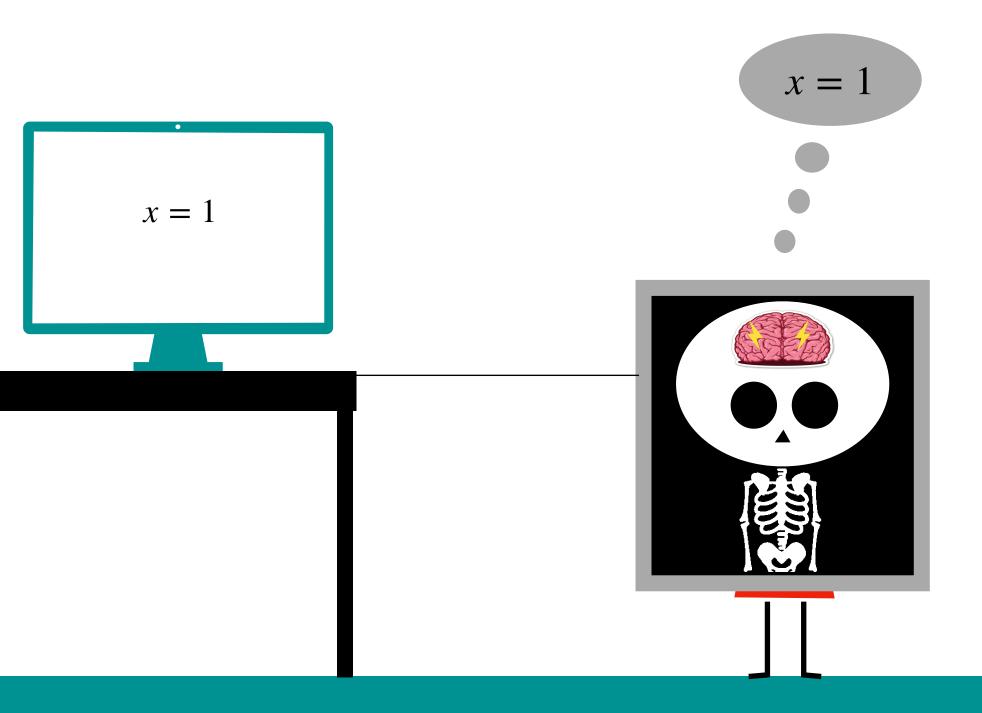
# **Power consumption attacks**

## <u>Power consumption attack:</u> the attacker knows the power consumption of the device executing the algorithm. He has access to « traces ».

## Many attacks as well

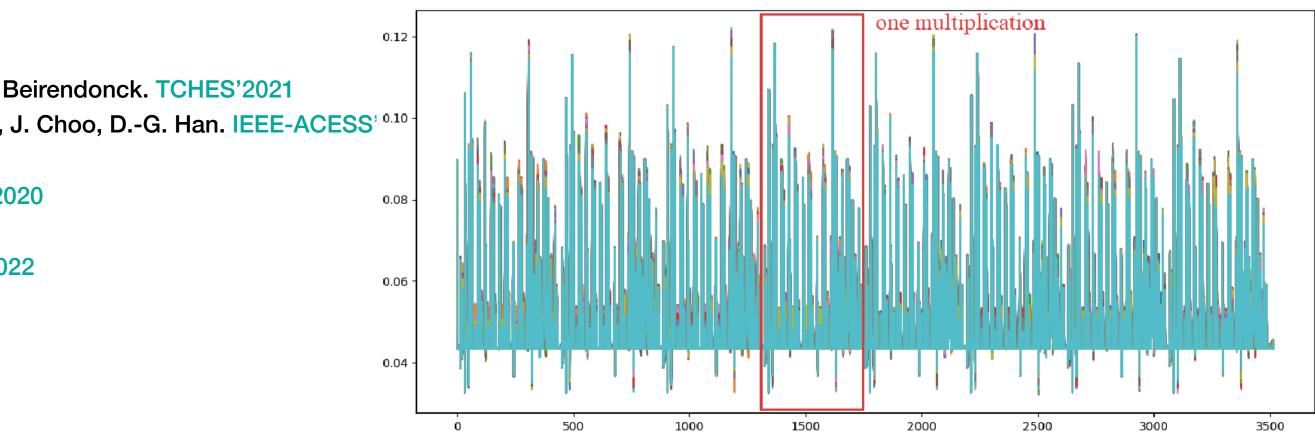
## An example presented in the next slides

- R. Primas, P. Pessl, S. Magnard. CHES'2017
- S. Bhasin, J.-P. D'Anvers, D. Heinz, T. Pöppelmann, M. Van Beirendonck. TCHES'2021
- B.-Y Sim, J. Kwon, J. Lee, I.-J. Kim, T. Lee, J. Han, H. Yoon, J. Choo, D.-G. Han. IEEE-ACESS<sup>10.10</sup>
- B.-Y. Sim, A. Park. eprint'2021
- P. Ravi, S. Sinha Roy, A. Chattopadhyay, S. Bhasin. CHES'2020
- E. Karabulut, A. Aysu. DAC'2021
- M. Guerreau, A. Martinellli, T. Ricosset, M. Rossi. TCHES'2022



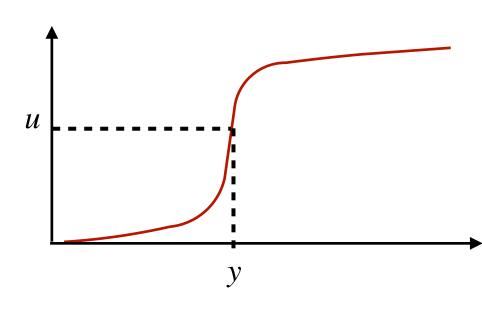
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**Usual suspects:** 

- multiplication with the secret: As
- NTT
- message encoding
- Fujisaki-Okamoto transform
- Internal distributions
- Cumulative Distribution Tables





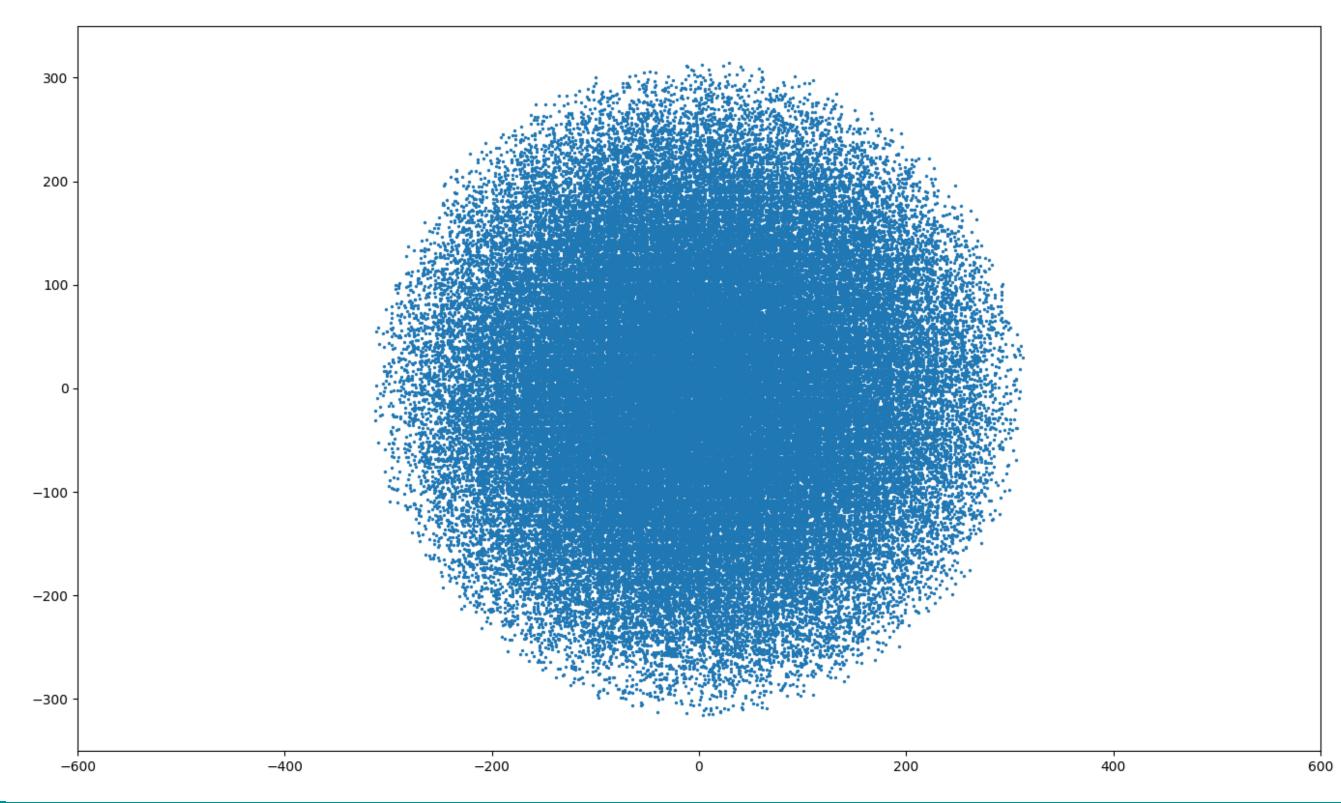


Signature algorithm:

- 1: compute c such that cA = H(m)
- **2:**  $\mathbf{v} \leftarrow \mathbf{a}$  vector in  $\Lambda(\mathbf{B})$  close to  $\mathbf{c}$

3: return  $s \leftarrow c - v$ 





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Falcon signature scheme

To avoid the hidden parallelepiped attack!

Take a close vector but not the closest.

Take the closest vector Add a Gaussian random shift  $z_0$ 

The distribution of signatures is then independent from the secret basis **B** 



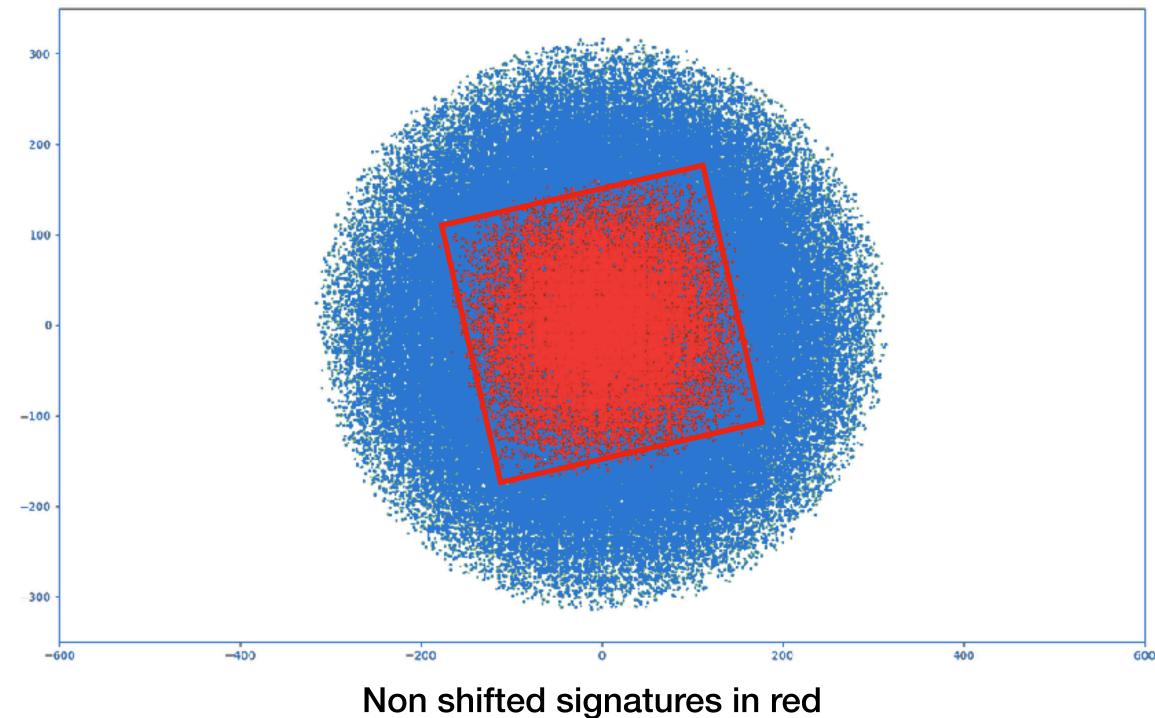




# Intuition of the power analysis attack of Falcon

Intuition of the attack

If we select the inputs such that the Gaussian shift is zero, we can apply the HPP attack.



What about high dimensions? There is a negligible amount of zero-shift in all 512 dimensions.

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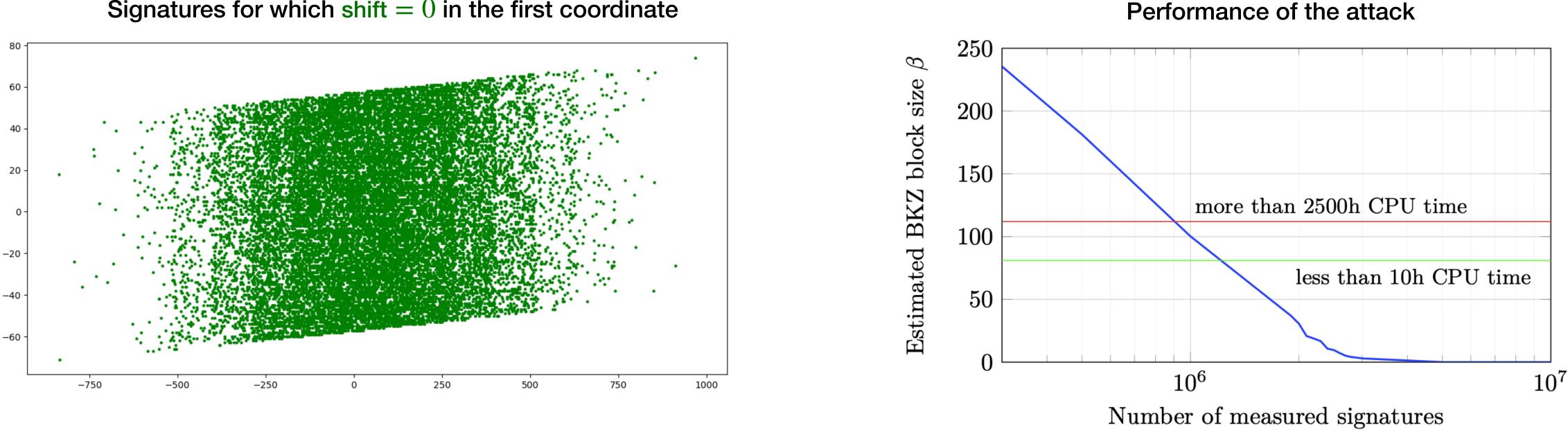
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# Single trace power analysis of Falcon

We focus on one dimension.

A single trace analysis can provide the information: shift = 0 or  $\neq 0$ .



Signatures for which shift = 0 in the first coordinate

It is possible to apply a partial HPP.

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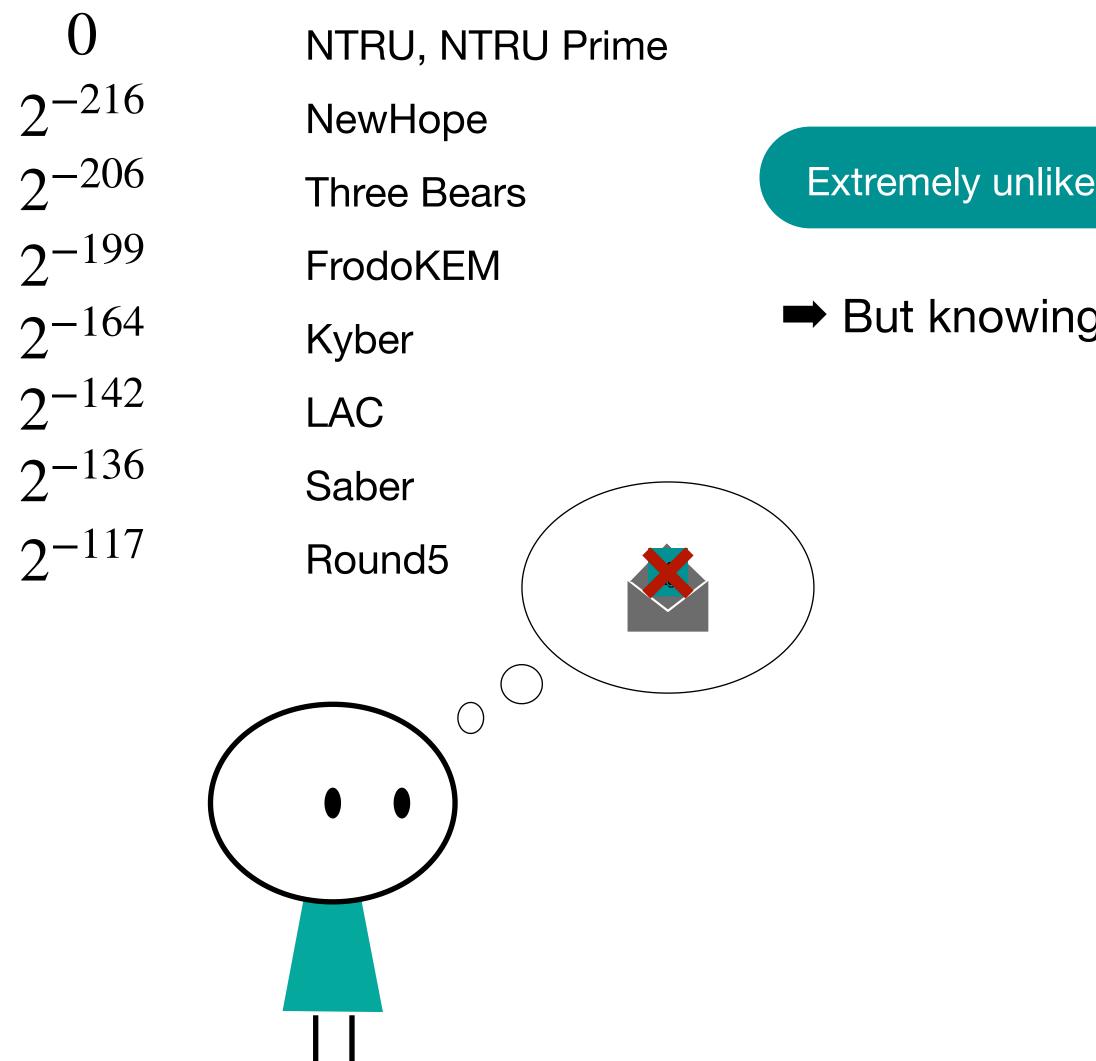
We recover one vector of the basis, this is enough to recover the full basis thanks to the structure of the private key.

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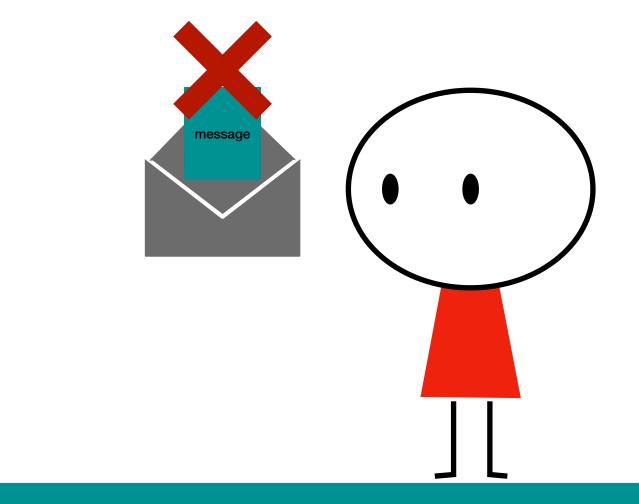
## Failure probability for a honest user



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Extremely unlikely for honest users

But knowing that a ciphertext has triggered a failure: is it a problem?

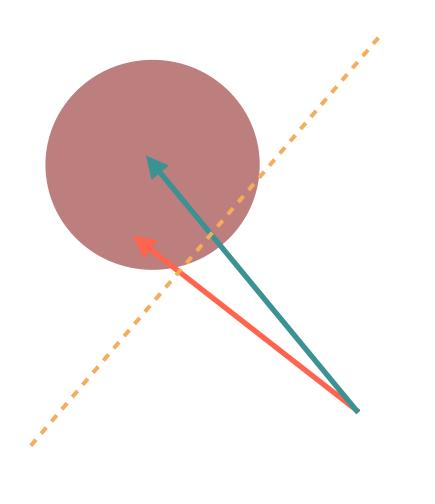






# What information do we gain from a decryption failure ?

Recall that 
$$m' \approx m + \left[\frac{2}{q}\left(\mathbf{e}^T \mathbf{z}' + \mathbf{e}'' - \mathbf{z}^T \mathbf{e}'\right)\right]$$



## Approximate hint with LeakyLWEestimator

► J.-P. D'Anvers, Q. Guo, T. Johansson, A. Nilsson, F. Vercauteren, and I. Verbauwhede. PKC'19

Dachman-Soled, L. Ducas, H. Gong and M. Rossi. CRYPTO'2020.

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$$\left\| \bigotimes \left\{ \frac{2}{q} \left( \mathbf{e}^{T} \mathbf{z}' + \mathbf{e}'' - \mathbf{z}^{T} \mathbf{e}' \right) \right\} \ge \frac{1}{2}$$

$$\left\| \bigotimes \left\{ \mathbf{s}^{T} \mathbf{w} \right\} \right\| \ge \frac{q}{4}$$

$$\left\| \bigotimes \left\{ \mathbf{s}^{T} \mathbf{w} \right\} \right\| \ge \frac{q}{4}$$

$$\left\| \bigotimes \left\{ \mathbf{s}^{T} \mathbf{w} \right\} \right\| \ge \frac{q}{4}$$

$$\left\| \bigotimes \left\{ \mathbf{s} = k \cdot \mathbf{w} + \epsilon \right\} \right\|$$

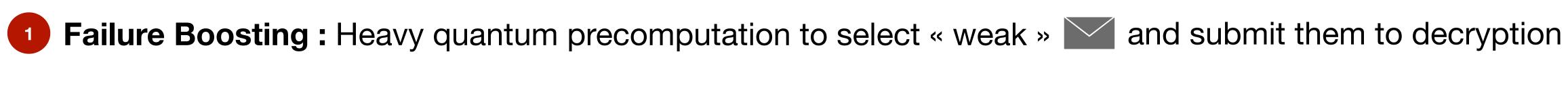
k and the standard deviation of  $\epsilon$  depend on

- the standard deviation of **W**
- the norm of S,  $\approx \sqrt{n\sigma}$
- the parameter *q*



# A generic decryption failure attack

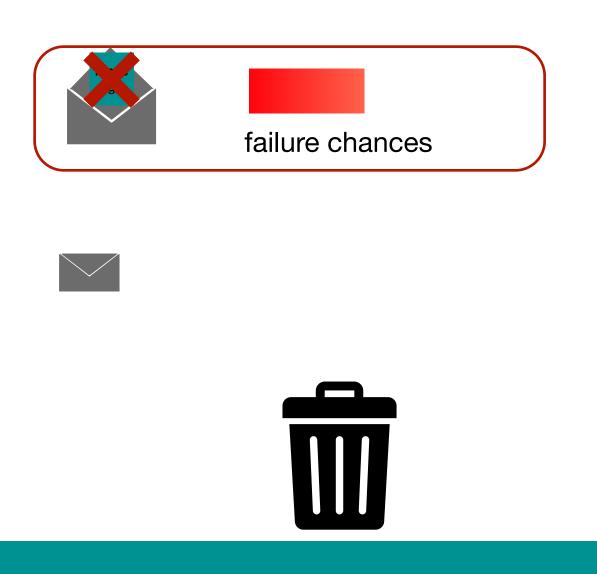
J.-P. D'Anvers, Q. Guo, T. Johansson, A. Nilsson, F. Vercauteren, and I. Verbauwhede. PKC'19





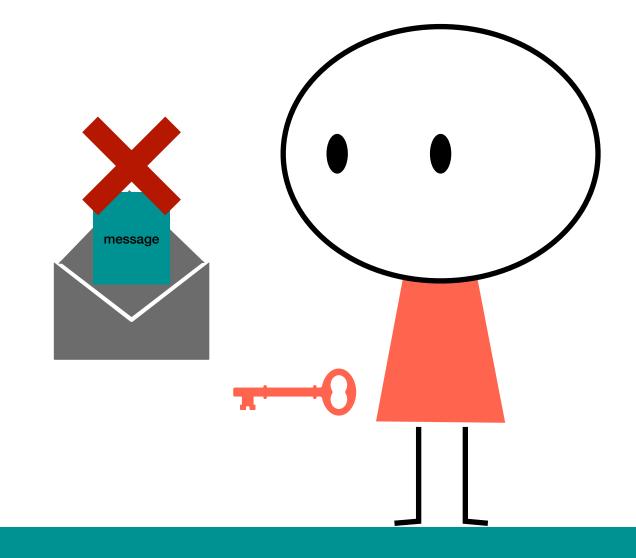
provides some information on z, e

## Found a weak ciphertext !



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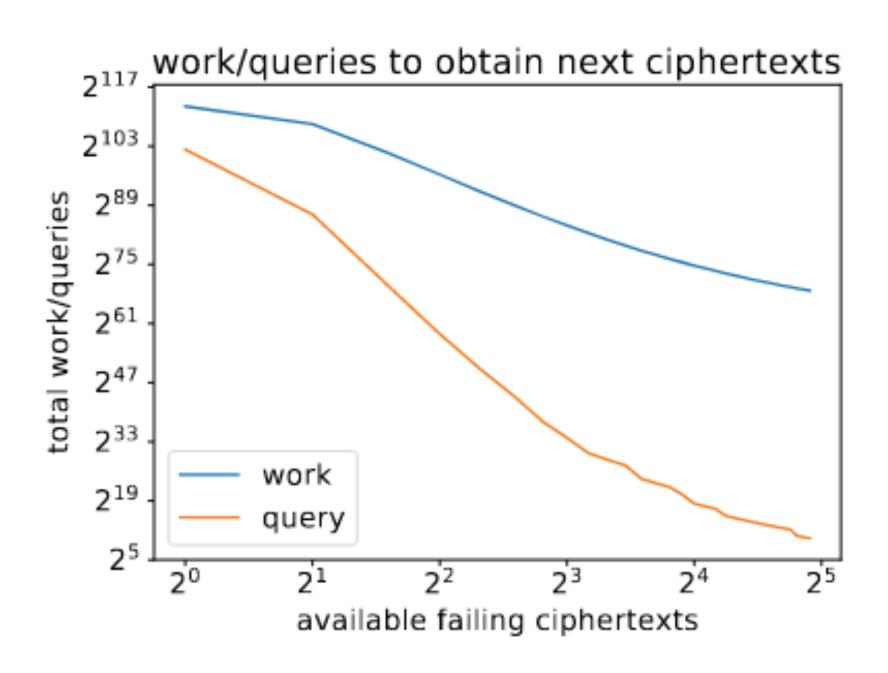






# Improvement: directional failure boosting attack

Geometric criterion for selecting new failures



## The cost is **dominated** by the search of the first failure

(One) failure is not an option:

*Bootstrapping the search for failures in lattice-based encryption schemes.* EUROCRYPT'2020. J.-P. D'Anvers, M. Rossi and F. Virdia.

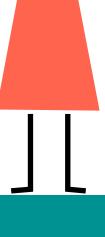


## Once one failure happen

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## the selection of the subsequent is easier.



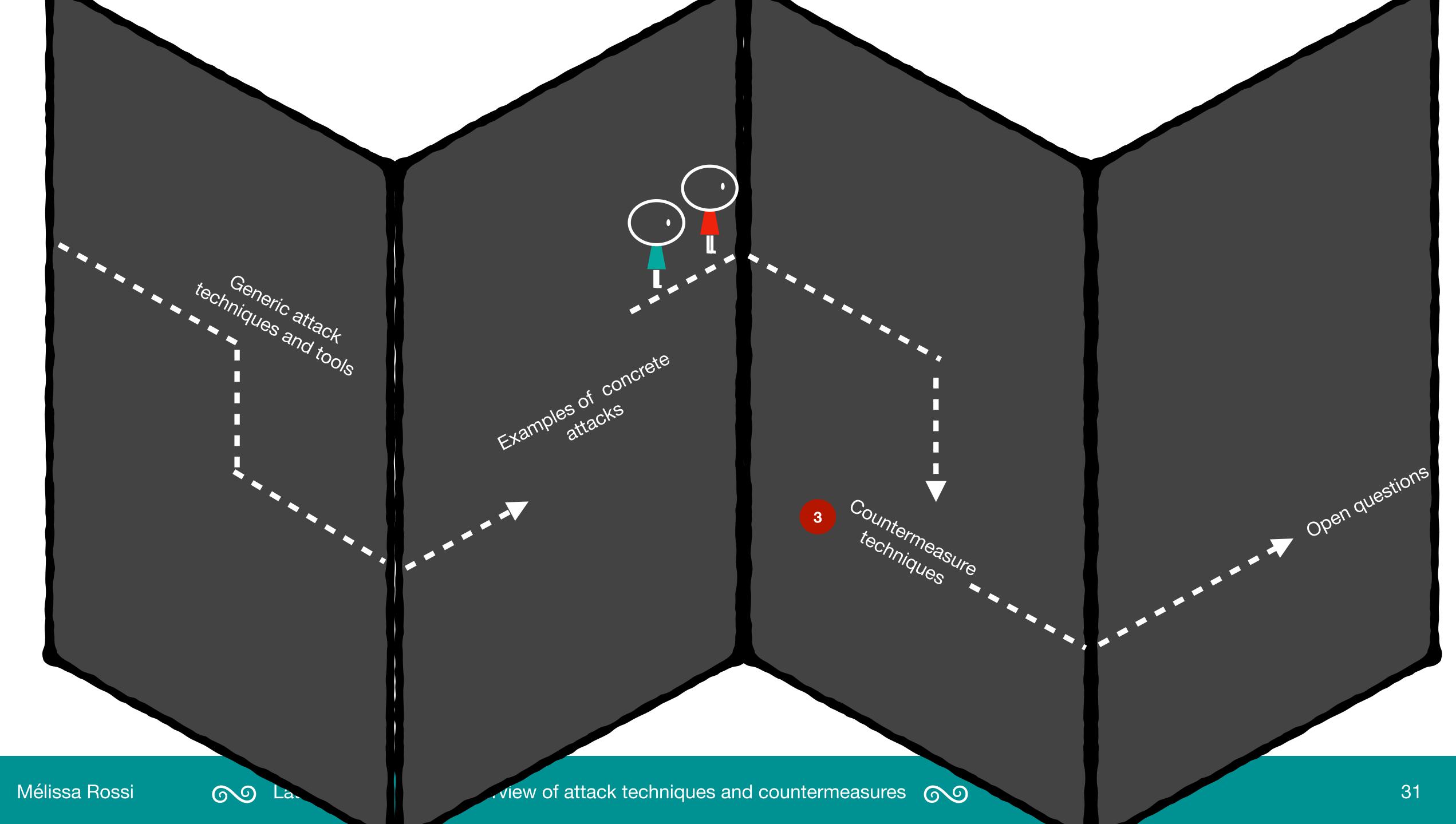
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ж

**S** =

e









The entry points include:

- computer-science unfriendly distributions like Gaussians.
- secret-dependent internal distributions.
- $\blacklozenge$  numerous operations with the secret.
- nonzero failure probability.

We want proofs of security!

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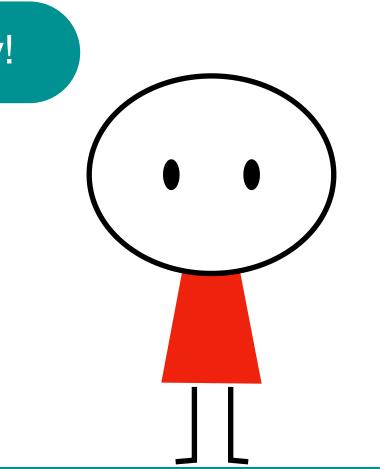
Here are some countermeasure techniques:

- For fixing the distributions:
- In a generic way:

Renyi divergence arguments

Polynomial approximations

Masking techniques adapted for lattices

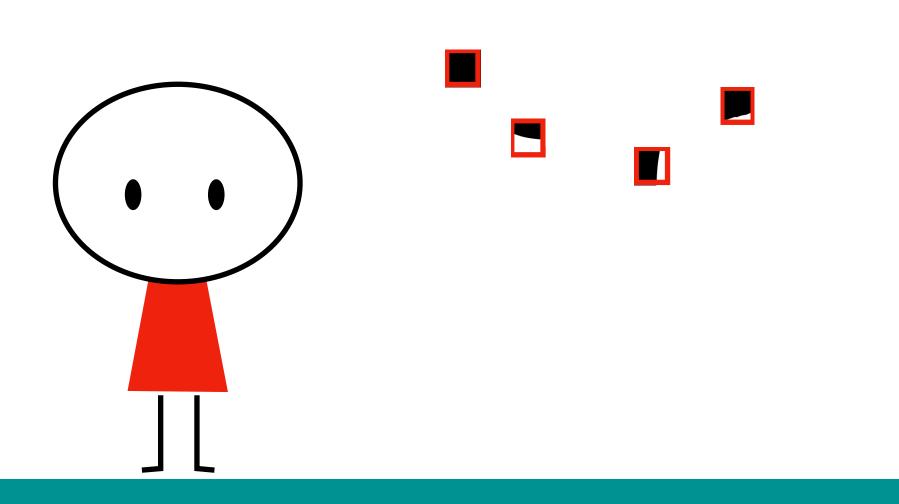




# 1) Rényi divergence arguments

S. Bai, A. Langlois, T. Lepoint, D. Stehlé, R. Steinfeld ASIACRYPT'15 T. Prest ASIACRYPT'17

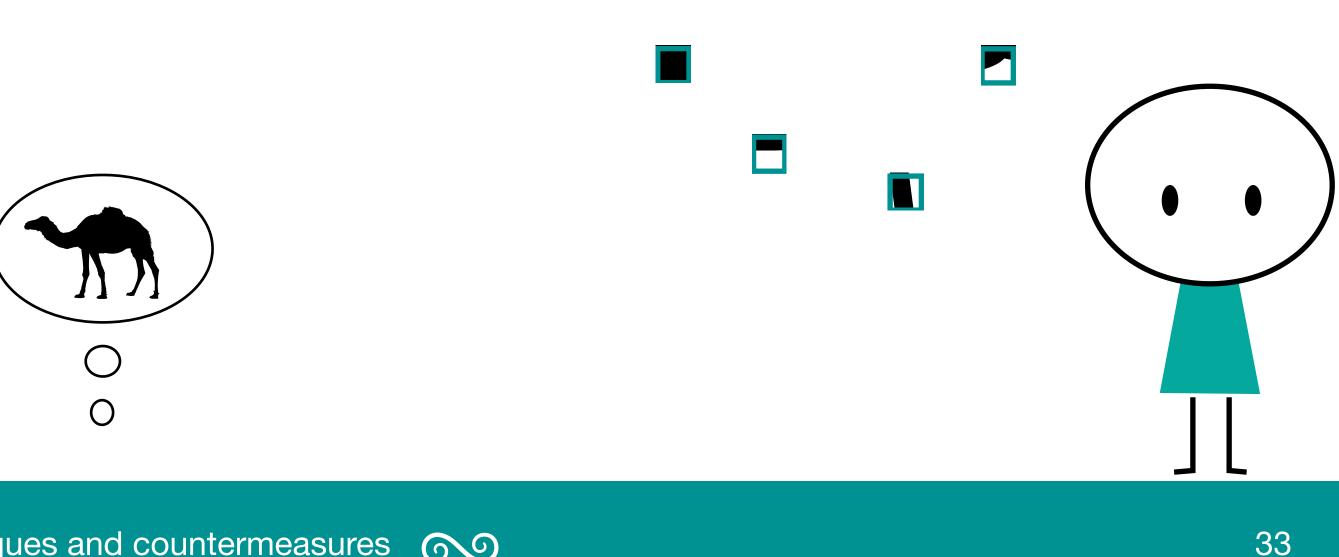
Distributions may be approximated/simplified because of the limited number of queries



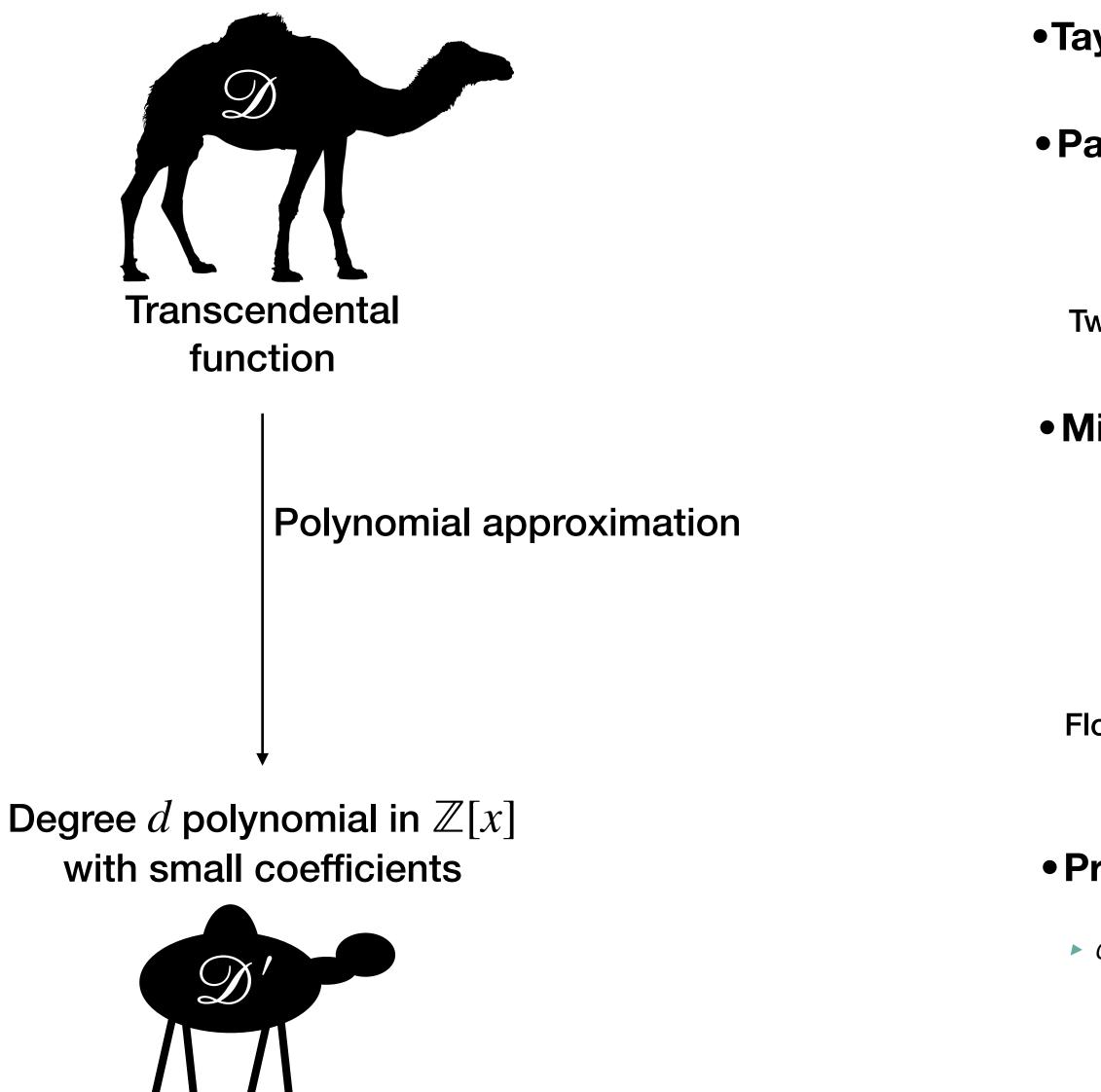
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- Take two cryptographic schemes
- One with distribution  $\mathscr{D}$
- One with an approximate distribution  $\mathscr{D}'$  with the same support
- Suppose that :
- **1.**  $\mathscr{D}$  and  $\mathscr{D}'$  are close enough :  $\left\| 1 \frac{\mathscr{D}'}{\mathscr{D}} \right\| \le 2^{-K}$
- 2. the number of sample queries is bounded
- Then, the bit security will remain almost the same.



# 2) Polynomial approximation for Gaussians



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aylor expansion 
$$\mathscr{D}'(x) = \mathscr{D}(0) + \mathscr{D}^{(1)}(0) \cdot x + \dots + \frac{\mathscr{D}^{(d)}(0)}{d!} \cdot x^d$$

• Padé approximants (rational function approximation)

► T. Prest ASIACRYPT'17

Two polynomials, higher degrees  $\mathscr{D}'(x) = \frac{P(x)}{Q(x)}$ 

## • Minimax computations : Sollya software package

- N. Brisebarre and S. Chevillard IEEE'07
- S. Chevillard, M. Joldes and C. Q. Lauter ICMS'10
- R. Zhao, R. Steinfeld and A. Sakzad IEEE'19

Floating point arithmetics

$$\mathcal{D}' = \arg\min_{\deg(P) \le d} \left( \sup_{x \in I} \left( 1 - \frac{P(x)}{\mathcal{D}(x)} \right) \right)$$

## Projections with respect to the Sobolev Norm

GALACTICS [...] ACM-CCS'2019. G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi.

$$\|f\|_{\infty} \le \sqrt{2} \cdot \|f\|_{S}$$



## Falcon Performance penalty factor :

► J. Howe, T. Prest, T. Ricosset and M. Rossi. PQ-CRYPTO'2020.

T. Pornin https://falcon-sign.info/falcon-impl-20190802.pdf

### Performance penalty factor : BLISS

G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi. ACM-CCS'2019.

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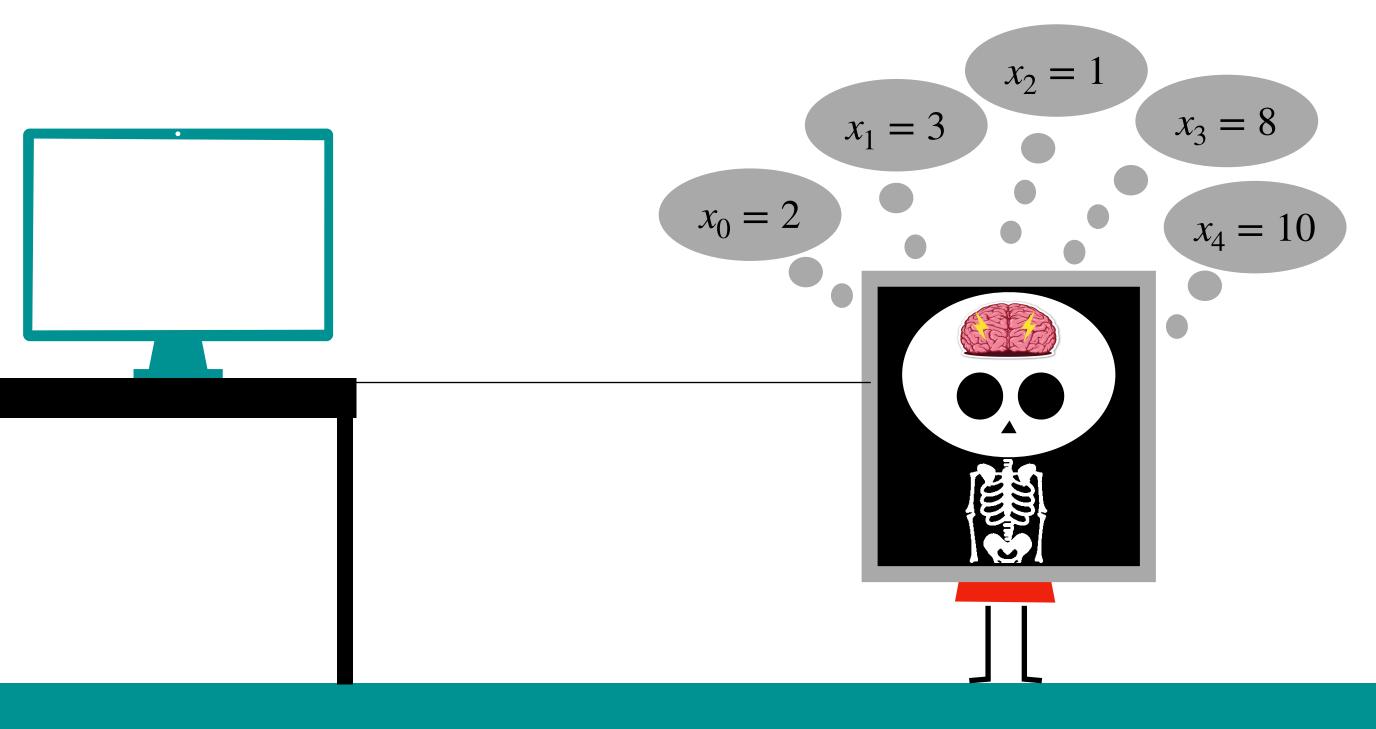




# Masking lattice-based schemes

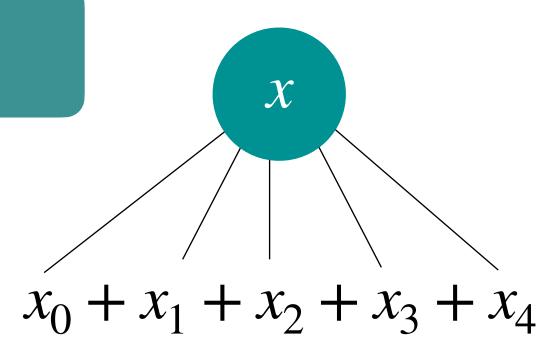
L. Goubin and J. Patarin CHES'1999 Designs for the multiplication of two shared values S. Chari, C. Jutla, J. Rao and P. Rohatgi CRYPTO'1999

Increase of the noise: Highly complicates the dependancies between the secret and the measurement



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Each share looks random. The only way to recover x is to know all of them. Masking order : d = 4.

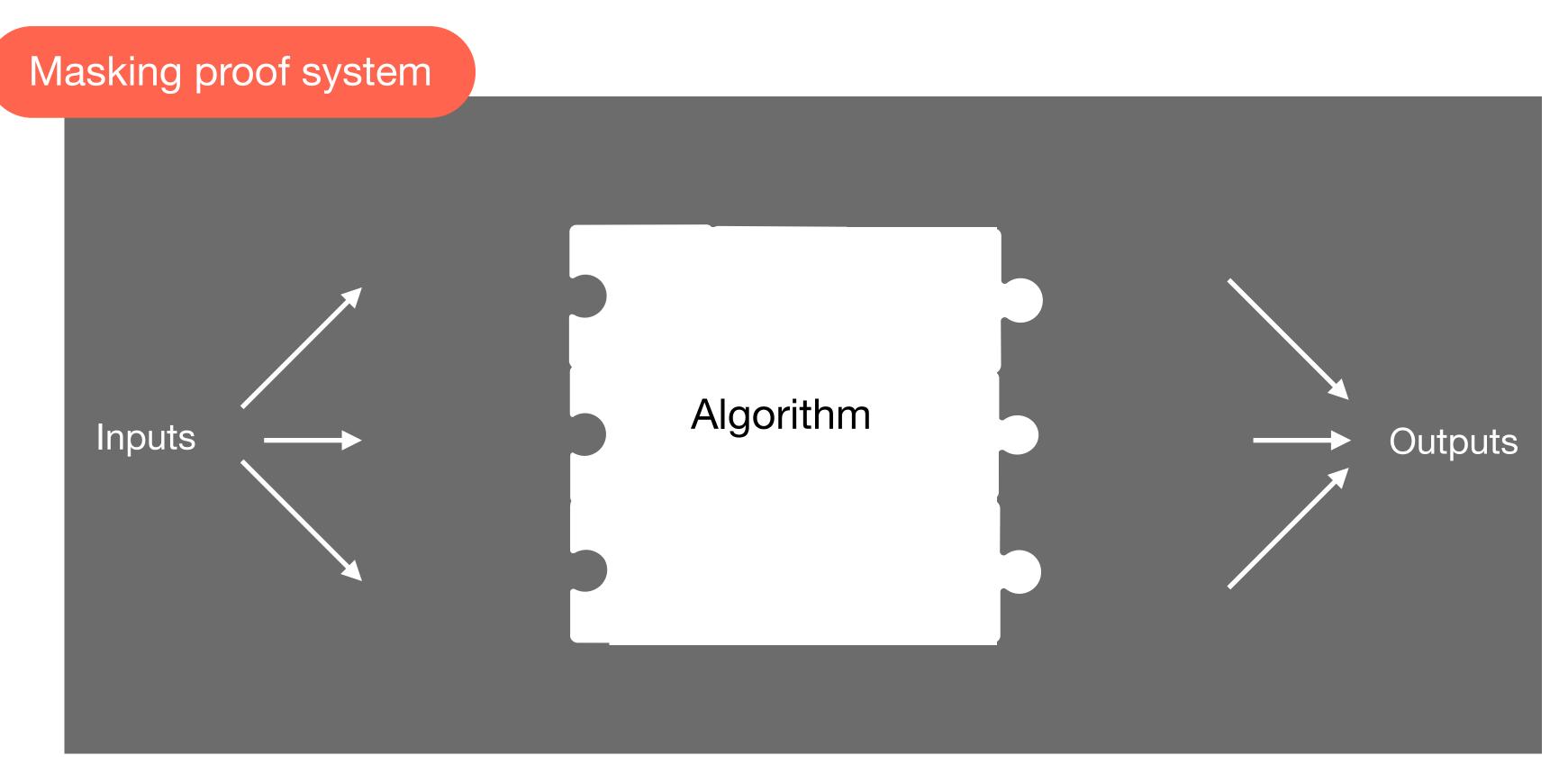
> The real secret value is x = 2 + 3 + 1 + 8 + 10= 24





# Masking lattice-based schemes

- Y. Ishai, A. Sahai and D. Wagner CRYPTO'2003



Proofs of masking for each gadget +Composition proofs

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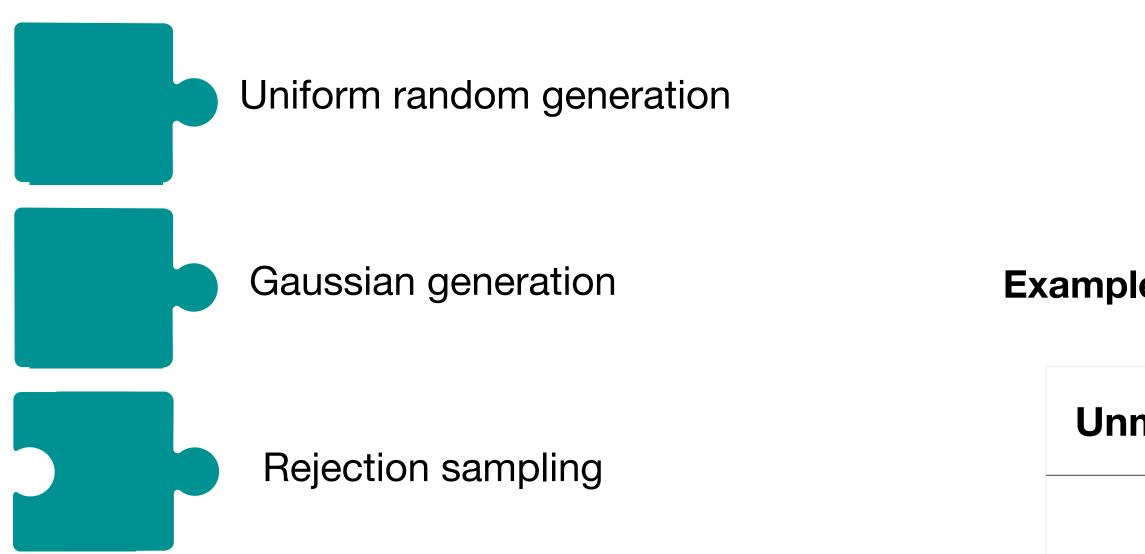
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• G. Barthe, S. Belaid, F. Dupressoir, P.-A. Fouque, B. Grégoire, P.-Y. Strub, and R. Zucchini. ACM-CCS'2016









- G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, B. Grégoire, M. Rossi and M. Tibouchi. EUROCRYPT'2017.
- G. Barthe, S. Belaïd, T. Espitau, P.-A. Fouque, M. Rossi and M. Tibouchi. ACM-CCS'2019.
- T. Espitau, P.-A. Fouque, F. Gérard, M. Rossi, A. Takahashi, M. Tibouchi, A. Wallet, Y. Yu EUROCRYPT'2022

## The constructions must use mask conversions

- ► J.-S. Coron, J. Großschädl and P. K. Vadnala CHES'2014
- J.-S. Coron, J. Großschädl, M. Tibouchi, and P. K. Vadnala FSE'2015
- ► J.-S. Coron CHES'2017

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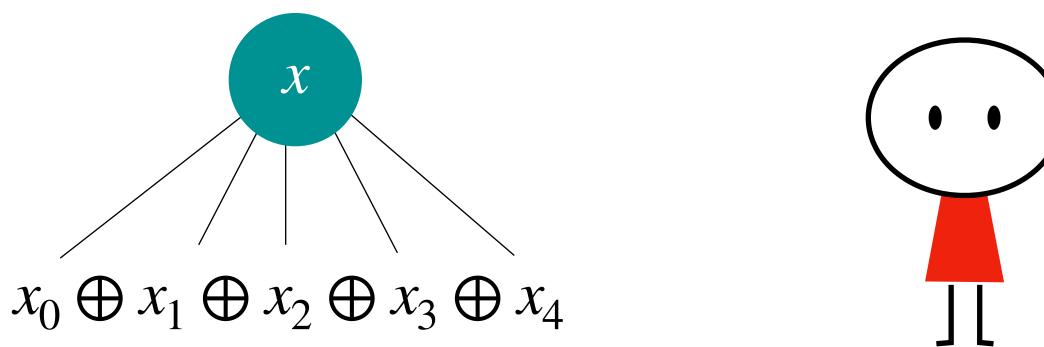
## Signature-adapted security property

Non-interference with public outputs

## Examples of overhead on the number of cycles for qTesla signature scheme

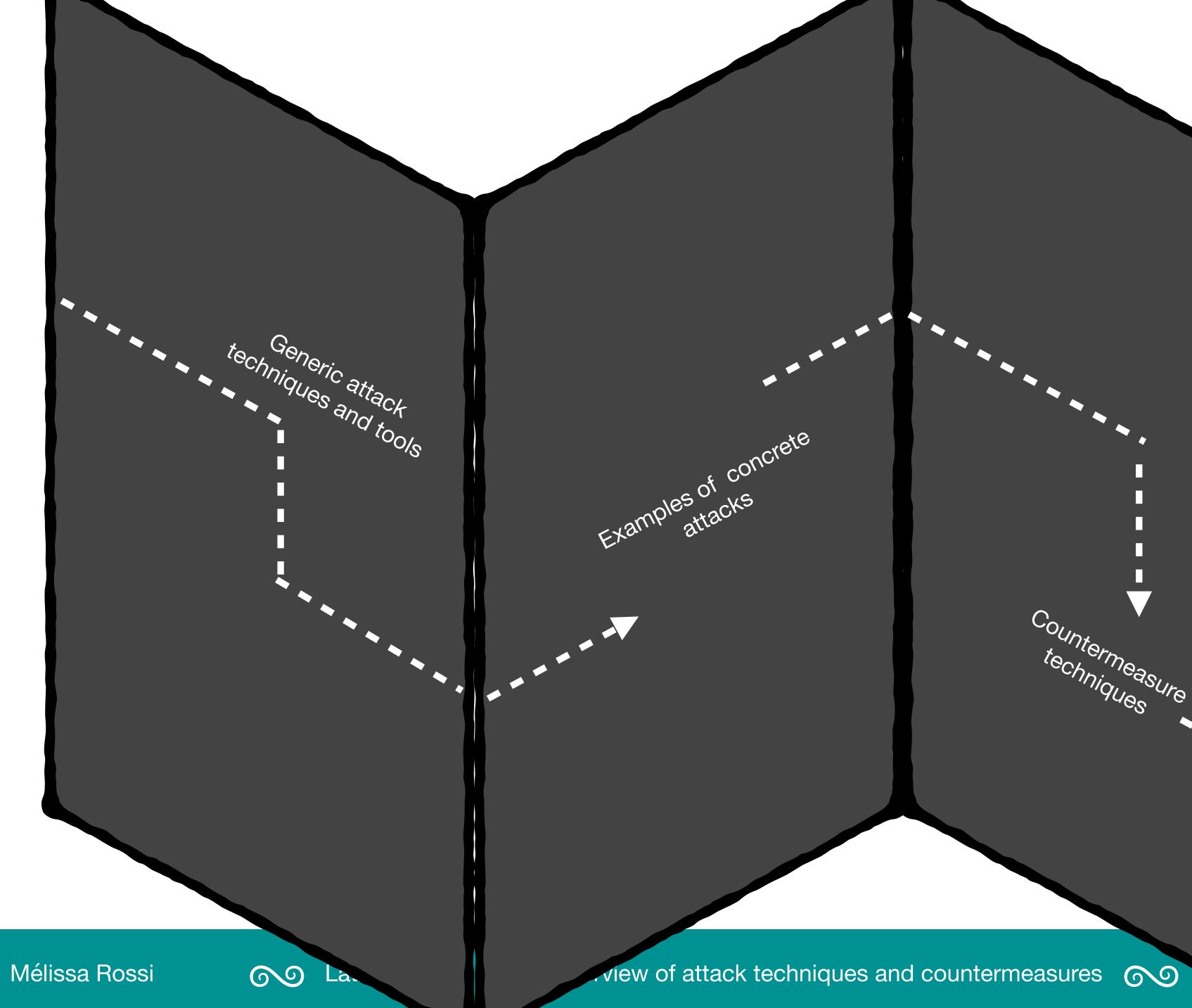
masked	Order 1	Order 2	Order 3	Order 4
1	$\times 4$	×21	× 37	× 60

F. Gérard and M. Rossi. CARDIS'2019.









Countermeasure techniques







## Masking friendly design

The designs contain many « masking unfriendly » features: Gaussian distributions, uniform small distributions, comparison of sensitive values, rejection, prime modulus...

Schemes designs that minimize the masking overhead at a cost of less efficient unmasked version.

## Other entry points

Are lattice-based schemes fault-resilient?

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# **Open problems**

## Fujisaki-Okamoto transform

This transform is needed because it protects against active attacks (IND-CCA2 security) but it highly increases the attack surface and introduces new attack entry points.

➡ Is re-encryption (or similar tests) inevitable? ➡ Is it possible to design a fully protected generic Fujisaki-Okamoto transform?

**Blackbox attacks** 

Cryptanalysis of ideal lattices









# **Des questions?**



## du vendredi 29 avril 2022 à 14h jusqu'au dimanche 8 mai 2022 à 18h.

moins de 25 ans? Vous pouvez être sélectionnés pour l'équipe de France

Plus d'infos sur : https://www.ssi.gouv.fr/agence/cybersecurite/france-cybersecurity-challenge-2022/

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## FRANCE CYBERSECURITY CHALLENGE



